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Effects of the quarantine on the individuals' risk of Covid-19 infection: Game theoretical approach

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Abstract In this study, we analyze the general or self-quarantine effects to the spread of the first wave of Covid-19 pandemic in the view of the game-theoretical approach. As in some other applications of game theory in different aspects of the literature, we focus on only the application of game theory to present the effects of quarantine during the three different stages -the start, the spread, the end- of the pandemic. We first choose three countries such as South Korea for self-quarantine, Italy, and Turkey for general quarantine during the analysis of the different stages of the spread. Then, we present a formula that will be an important tool for the creation of the payoff matrices and give the general procedure for the creation of the payoff matrix for each stage of the pandemic process. After that, we generate the payoff bimatrix for each stage of the pandemic by using the average of the daily diagnosis number/number of tests for each country. Moreover, we try to find the optimal strategy of the game. Additionally, to determine the necessity of the continuity of the quarantine, we use the repeated game approach in our analysis, as well. Therefore, we convert the game only for the spread stage to the repeated game for each country. Finally, we obtain the Nash equilibrium of all games for each level of the pandemic. The results show that the quarantine has important effects to be infected or not, and the spread of the pandemic at each level. In addition to these analysis results, we compare the death rates of the considered countries and show that the results are almost parallel to that are obtained for the quarantine requirement of each country by game-theoretical approaches.

1. Introduction

Game theory can briefly be defined as a research branch that studies decision-making processes in conflict situations [1]. We can also define game theory as a branch of science that deals with conflict situations with a mathematical approach [2]. This theory is emerged as a result of the application of mathematical approaches to some situations in World War

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II [3]. The first book written about the game theory in detail was written by Von Neumann and Morgenstern in 1944 [4].

Game theory has developed over time and has found application in almost all branches of science. Some examples of these practices can be listed as follows: Haywood, in 1954, developed an analogy between the current military doctrine and the game theory presented by von Neumann using the two war decisions in World War II [3]. In 1962, Borch addressed the problem of determining the correct premium rates for subgroups of an insurance collective using game theory [5]. Agrawal and Heady applied the game theory models to decision-making processes under uncertainties in agriculture in 1968 [6]. In 1971, Snyder examines a game-theoretical model of disarmament of the United States and the Soviets from two different perspective [7]. In 1979, Lee et al. examined the coalition in the elections held in France in 1951 as a cooperative game of n-person [8]. Fisk examined the relationships between two game theory models (Nash uncooperative games and Stackelberg games) and some problems in transportation systems modeling. He used companies competing for intercity passenger travel and signal optimization problems as examples in his study in 1984 [9]. In 1996, Yeung developed a differential game model for interchangeable market products [10].

In 2005, Aumann and Schelling developed the relationship between competition and cooperation with the help of game theory and were awarded the Nobel Prize for their work [11]. Doebeli and Hauert investigate some game-theoretical model of cooperation and review the cooperational situations, multiperson interaction [12]. In 2010, Madani presented the applicability of Game Theory to water resource management and conflict resolution through a series of non-cooperative water resource games [13]. Moreover, in 2010, Chlebik investigates terrorism from the terrorist's perspective and presented some matrix game model in his study [14]. In 2014, Young demonstrated that the costs incurred in the process of a region becoming a sovereign state may be large enough to outweigh the long-term financial benefits of independence using game theory [15].In the same year, Köse presented a model of international sanctions by using matrix games [16]. With the help of Game Theory, Diesen addressed the rise of the 'inter-Democratic' security institutions after the collapse of the Soviet Union in 2015 and its implications for their relations with Russia [17]. In 2015, Erkut made a new and original contribution to the literature on the Annan Plan and the Cyprus conflict, providing a better understanding of the pre-referendum political conditions using cooperative game theory [18].

Farhidi and Madani analyzed the conflicts related to Iran's nuclear program using game theory [19]. Little examined in 2017 how non-competitive elections affect citizen well-being compared to a non-electoral baseline from a point of view he developed with his game theory approach [20]. In 2019, Caleiro et al. studied global development and climate change with game theory [21]. Moldalieva and Heathershaw demonstrated the role of the elite in the struggle for transparency and accountability in Kyrgyzstan's mining sector in their study in 2020 and included game theory in their work [22]. In the same year, Izgi and Özkaya demonstrated the necessity of agricultural insurance with the MN-approach, in [23], used in game theory [24]. As can be seen from the above examples, the use of game theory has improved a lot since its first day.

On the other hand, the Coronavirus disease 2019 (Covid-19) raised in Wuhan, China in 2019 and swiftly spreads to other cities in China in a couple of days. Then, it was declared as a public health emergency by WHO on 30.01.2020. Due to late precautions taken by China, the infected people carried the virus all around the country and went to other countries. The red alert is given all around the globe and all countries started to take some precautions such as stopping the international flights, closing their borders with neighbors, and declaring regional or national lockdown. The medical researchers try to understand the virus while the scientists focused on modeling the expansion by using mathematical tools. Some studies about the mathematical model of the spread are as follows: Ivorra et al.presented a mathematical model by investigating the data provided from China and World Health Organization [25]. Kurcharski and his colleagues used a stochastic transmission model by using the data of Covid-19 in Wuhan and international cases and they estimated the variation of the transmission during January and February [26]. Hellewell et al. developed a stochastic transmission model for understanding the isolation and contact tracing effects on Covid-19 cases [27]. Hu et al. proposed an artificial intelligence method to forecast the end time of the virus across China [28]. Cakir and Savas present a new and modified mathematical model depending on the number of the patient at any time t for Covid-19 pandemic in Iran. They concluded that the personal and public precautions should have taken immediately [29]. Wang et.al investigate some mathematical models for the Covid-19 pandemic in their paper. They also introduce some basic models and state the fundamental notations and framework for the epidemiological modeling [30]. Caparros and Finus discussed the personal and coordinated precautions across the countries affected by Covid-19 in the perspective of a weakest-link public good game. In their results, they conclude that regional or global cooperation is important during the pandemic [31]. Atangana and Araz investigate the Covid-19 pandemic in Turkey and South Africa in detail. They used statistical tools in order to make an analysis of the data gathered from Turkey and South Africa. Moreover, they present a new mathematical model for Covid-19 consisting of nine classes. Furthermore, they examine the reason for the number of deaths and infected people in Turkey and South African. In addition to this, they explain why there are fewer deaths and cases in South Africa comparing to Turkey [32]. Araz study a mathematical model for the spread of Covid-19. She presents the stability analysis and the optimal control system. Then, the model is developed to the non-local operator for all cases and, the positiveness of the system solutions is stated. Furthermore, some other applications are investigated under different situations and solved numerically [33].

Matjaz et al. present a simple iterative method in order to guess the Covid-19 cases under some assumptions such as governmental data is reliable [34]. Momtazmanesh et.al present some conclusions about the Covid-19 outbreak. They present the result that the Covid-19 pandemic is a test for our preparedness at the national and international level. The crucial thing is cooperation locally or globally during pandemics. The precautions should be taken immediately and the situation should be declared as soon as any kind of pandemic arises [35]. Hancean et al. study the very beginning of the Covid-19 pandemic in Romania and the human-to-human transmission networks. They analyze the time dynamics and the structural characteristic of these transmission networks [36]. In 2021, Atangana and Araz study comprehensively the analysis of stochastic differential equations with the global derivative with integer and non-integer order. They obtain numerical solutions for these classes and make the error analysis. They investigate some epidemic problems such as zika virus spread model, Ebola model, and zombie virus spread model in order to illustrate the application of these operators. They solve these problems numerically using the proposed scheme. They conclude that the real-life problem containing higher complexity can be explained by the help of these operators [37]. The number of studies about the modeling and understanding of the Covid-19 virus rapidly increases over time.

In this paper, we investigate the effects of the general or self-quarantine on the individual risk of Covid-19 infection by the game-theoretical approach. Since there are different usage of the game theory in the literature, we mainly study the application of game theory to the risk of infection during the first wave of the pandemic in order to show the effects of quarantine. By using the idea of the matrix games in the game theory, we analyze three different countries, that are South Korea, Italy, and Turkey, in the time period between February and May of 2020. The main reason for these selections is that S.Korea nearly ended the spread without a lockdown (self-quarantine, i.e. staying at home without any obligation). Additionally, Italy had serious trouble and became the center of Covid-19 in the world for a while even though they announced a general quarantine. Lastly, Turkey handled the situation well with general quarantine and praised by WHO for this reason [46].

We examine the effects of the quarantine on infection for the different stages of each country during the spread of the first wave of Covid-19 pandemic, separately as the start, the spread, and the end. To do so, we first obtain the data belonging to these countries from some resources confirmed by WHO. Then, we create the payoff bimatrix for each stage of the pandemic by using the average of the daily diagnosis number/number of tests for each stage of the pandemic for these three countries. People sometimes may violate the quarantine since the quarantine is a long term process, although they generally try to keep it. Under this violation, we prefer to use a repeated game approach in order to analyze the risk of infection. For instance, we convert the matrix games in the peak stage of each country to a repeated game. A similar approach may be done for the other stages of the pandemic. Then, we reveal the Nash equilibrium of all games of three countries for each level of the Covid-19 pandemic.

The remainder of the paper is organized as follows. In Section 2, some theoretical background used throughout the paper is briefly presented. Then, the payoff matrix creation for the pandemic process' analysis is explained. In the third section, the effect of the quarantine on the spread of the Covid-19 pandemic for South Korea, Italy, and Turkey is comprehensively presented in game theory. The last section concludes the study.

2. A game theory approach for covid pandemic process

In this section, as a starter, we give some definitions which is used throughout the Covid-19 pandemic process' analysis. Then, we present a formula that will be an important tool for the creation of the payoff matrices. Later, we present the steps of the payoff matrix creation with the details. Finally, we analyze the results obtained by game-theoretical approaches.

Definition 1. (Strategic Form of a Game [38]) The strategic form or normal form of a two person game is given by $u_1(x,y)$ and $u_2(x,y)$ are the real valued functions over $X \times Y$, representing the payoff of the players, where X and Y are the pure strategy sets of the player. If $u_1(x,y) \neq u_2(x,y)$, then the game is called a nonzero sum game

Definition 2. (**RepeatedGame**)If a game is played in a row, the entire game is called a repeat game. If the repetition number of the game is finite, then the game is called finitely repeated game. If the game is played infinitely many time, then the game is said to be infinitely repeated game.

Definition 3. (Nash Equilibrium [39]) If a strategy is the best response to other strategies, the binary that these strategies form is called pure Nash equilibrium.

In addition to these definitions, we state and prefer to use the following average formula to summarize and reduce the data for each stage of the pandemic into the applicable form of the bimatrix game since we relatively determine the stages of the pandemic according to the daily data for each country.

$$p = \frac{1}{n}\sum_{i=1}^{n}\frac{d_i}{t_i} \tag{1}$$

Here,

p: The average of the probability of a person being infected.

d_i: The number of the infected person whose PCR test is positive in the i^{th} day.

t_i: The number of PCR test which is done in the i^{th} day.

n: The number of the day in the stage (i. e. the beginning, peak or slow-down) of the pandemic.

p': The average probability of a person not being infected is $p' = 1 - p$.

Next, we generate the payoff matrix for each stage of the pandemic for each country by using the quantitie(s) obtained by Eq. (1) for the payoff matrices in our analysis. In the lights of these information and the following assumption and notation, we create the entries of the payoff matrix.

The general procedure for creation of the payoff matrix:

1. The player strategy set consist of two elements as $S = \{Q, Q'\}$ where Q: Keeping the quarantine and Q': Breaking the quarantine.
2. The payoff for the strategy Q is p' and Q' is $-p$, which are obtained by Eq. (1), since p' is a positive option that means it is the action keeping the player safe. The breaking quarantine affects the person negatively therefore we use minus sign for the value of average probability of a person being infected to present this effect, $-p$, explicitly.
3. If the both players choose the action Q, which is (Q,Q) strategy pair, they both get the payoff p' since they select to prevent themselves from the infection. Equivalently, they both get (p',p') as their payoff.
4. If one of the players choose Q', in other words one selects to break the quarantine, i.e. (Q,Q') or (Q',Q) strategy pair, the player chooses Q' strategy increases the risk of infection

so that his/her payoff decreases and the payoff belong to Q' is $-p$. Therefore, their payoffs are $(p', -p)$ and $(-p, p')$ regard to (Q, Q') or (Q', Q) strategies, respectively.
5. If the both players decide to break the quarantine, they both increase the risk of infection and their payoffs also negatively double since they both are out.

Under the favor of the above steps, we create the payoff matrix as,

$$CountryName_{Stage} = \begin{bmatrix} & Q & Q' \\ Q & (p', p') & (p', -p) \\ Q' & (-p, p') & (-2p, -2p) \end{bmatrix}$$

where Q: Keeping the quarantine and Q': Breaking the quarantine.

Finally, we perform game theoretical tools for the case analysis of the pandemic in South Korea, Italy and Turkey in order to explain and understand, respective, the self or general quarantine effects during to Covid-19 pandemic in the first wave.

3. Case Studies

3.1. Case I: South Korea

In this section, we analyze the effects of self-quarantine during the first wave of the Covid-19 pandemic in South Korea. Throughout the analysis of South Korea, we make use of the data announced from 01.02.2020 to 09.04.2020 even though there are some Covid-19 cases diagnosed before these dates which are negligible and does not affect the analysis, since the governments all around the world did not update their statistics regularly in the very beginning of the spread [40,41].

In the first stage, we take account and analyze the data during 1–25 February since the average of the numbers of test and diagnose in daily base are small. We name this stage in the time period, 1–25 Feb., as the start. In order to create the payoff matrix, we follow the procedure of the payoff matrix- creation given above: Step (1). There are two strategies Q and Q' which is valid for all stages, stay in quarantine and break the quarantine, respectively. Step (2). We first evaluate the average of the probability of a person being infected and the average probability of a person not being infected as are $p = 0.0160$ and $p' = 1 - p = 0.9840$, respectively, by Eq. (1). Step (3–5). We construct the payoffs for the strategies $(Q, Q), (Q, Q'), (Q', Q)$ and (Q', Q'). Later, we generate the payoff bimatrix SK_{start} by using the payoffs p and p', where SK denotes South Korea. Then, we get the following payoff matrix for the first stage,

$$SK_{start} = \begin{bmatrix} & Q & Q' \\ Q & (0.9840, 0.9840) & (0.9840, -0.0160) \\ Q' & (-0.0160, 0.9840) & (-0.0320, -0.0320) \end{bmatrix}$$

where Q: Keeping the quarantine and Q': Breaking the quarantine.

In order to see the gain/loss of the players, we find Nash equilibrium point of this game as known in the literature: First we put a mark on the maximum value of each column in the first entry of the binary. Then, we do the same for the maximum value in each row of the second entry in the binary [42]. The binary which has marks on each entry becomes the Nash equilibrium points. In this case,

$$SK_{start} = \begin{bmatrix} & Q & Q' \\ Q & (\widetilde{0.9840}, \widetilde{0.9840}) & (\widetilde{0.9840}, -0.0160) \\ Q' & (-0.0160, \widetilde{0.9840}) & (-0.0320, -0.0320) \end{bmatrix}$$

Finally, we see that there are two marks on the first binary of the bimatrix that is $(0.9840, 0.9840)$. In other words, the Nash equilibrium of this game is (Q, Q) strategy. It means that it is the best outcome for each player. Therefore, we conclude that staying in the quarantine during very beginning of the spread is the best option for each player. On the contrary, it is clear that the players have the worst outcome in the (Q', Q') strategy as $(-0.0320, -0.0320)$. It is obvious that, (Q', Q') strategy is definitely not worth to play.

As the second stage, we analyze the time period when the average of the test numbers and diagnoses reach the peak, that happens between the dates 26.02.2020 and 11.03.2020. We refer this period as the spread. Similar to the previous case, we create the payoff bimatrix by follow up the payoff matrix creation steps for this stage as:

$$SK_{spread} = \begin{bmatrix} & Q & Q' \\ Q & (0.9634, 0.9634) & (0.9634, -0.0366) \\ Q' & (-0.0366, 0.9634) & (-0.0732, -0.0732) \end{bmatrix}$$

The Nash equilibrium point is $(0.9634, 0.9634)$, that is (Q, Q), for the spread stage. Even though the risk of the getting virus is increased, it is still best choice to stay inside. On the other hand, it is very important to notice that the loss of each player in (Q', Q') strategy is doubled comparing with the value in the start stage. Therefore, we may say that it becomes more risky to go out during the dates of peak.

As the last stage, we use the data from 12.02.2020 to 09.04.2020 the situation of the slowing down of the spread and we name this stage as end. In other words, we investigate the case during the government takes control over the spread. The payoff bimatrix is similarly generated as follows:

$$SK_{end} = \begin{bmatrix} & Q & Q' \\ Q & (0.9870, 0.9870) & (0.9870, -0.0130) \\ Q' & (-0.0130, 0.9870) & (-0.0260, -0.0260) \end{bmatrix}$$

We see that the Nash equilibrium point implies the same result that is staying inside with the payoffs $(0.9870, 0.9870)$. Moreover, we observe the loss of players in the breaking quarantine strategy (Q', Q') is decreased comparing to the spread stage. However, it is still clearly the worst option to be chosen by the players.

In order to avoid the repetition of the application, we skipped to analyze the third stage of the pandemic for Italy and Turkey. The same analysis can be done by using the same manner in the third stage of South Korean case.

Furthermore, we examine the situations if the players have to make selection again, for instance, that is the players have to make a choice during the spread stage again. In other words, we consider the game as the repeated game in this situation Fig. 1. Here, we give the game tree of the spread stage as below, (see for the repeated game and the game tree in details [43]).

After that, we generate the payoff bimatrix for the second round of the game by using the values in the game tree as,

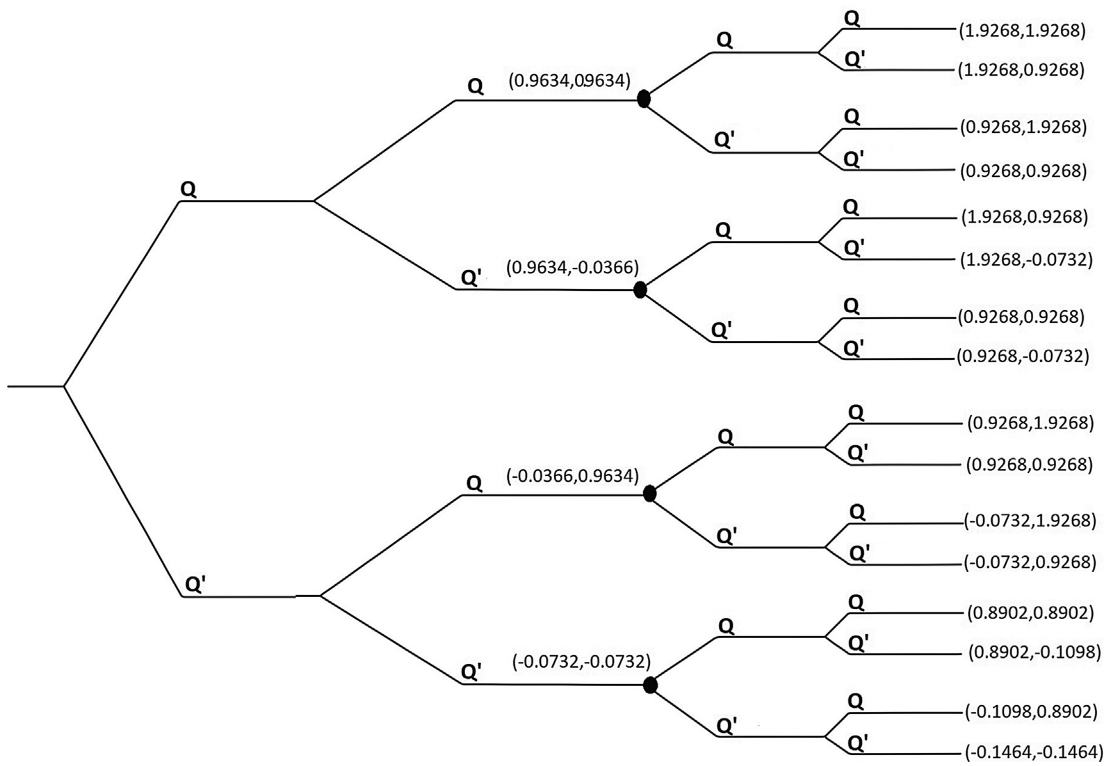

Fig. 1 Game tree for the second round for the spread.

$$SK_{Round\_II} = \begin{bmatrix} (\overline{1.9268}, \widetilde{1.9268}) & (\overline{1.9268}, 0.9268) & (\overline{1.9268}, 0.9268) & (\overline{1.9268}, -0.0732) \\ (0.9268, \widetilde{1.9268}) & (0.9268, 0.9268) & (0.9268, 0.9268) & (0.9268, -0.0732) \\ (0.9268, \widetilde{1.9268}) & (0.9268, 0.9268) & (0.9202, 0.8902) & (0.8902, -0.1098) \\ (-0.0732, \widetilde{1.9268}) & (-0.0732, 0.9268) & (-0.1098, 0.8902) & (-0.1464, -0.1464) \end{bmatrix}$$

The Nash equilibrium point is $(1.9268, 1.9268)$ which is the value of the strategy (Q, Q). Therefore, it can be seen that the player payoffs increased if they stay in the quarantine when the players have to make selection again. However, if the players go out repeatedly, their loss gets doubled, $(-0.1464, -0.1464)$ in this case. In other words, the probability of being infected doubled. Consequently, if the number of the play increases, the importance of the quarantine effects reflect more explicitly and also the loss and profit increase, relatively.

3.2. Case II: Italy

In this section, we investigate the effects of the quarantine during the first wave of Covid-19 pandemic in Italy as it is analyzed for South Korea. In order to do the investigation, we use the data from 21.02.2020 to 14.04.2020 [44]. The time period between 21.02.2020 and 06.03.2020 is named as the start. In addition to this, the spread got faster starting from 07.03.2020 until 14.04.2020 according to the data provided in [44] so that we refer this time period as the spread.

We use the same approach as it is done for the analysis of South Korea, and we begin the analysis with the first stage. We follow up the procedure given above in order to create the payoff matrix for the start case. The possibility of infection and not getting infected by the virus are evaluated as 0.1142 and 0.8858 by Eq. (1), respectively. Then, we obtain the payoff bimatrix of the start case as:

$$It_{start} = \begin{bmatrix} & Q & Q' \\ Q & (0.8858, 0.8858) & (0.8858, -0.1142) \\ Q' & (-0.1142, 0.8858) & (-0.2284, -0.2284) \end{bmatrix}$$

We find the Nash equilibrium point as $(0.8858, 0.8858)$, that is, (Q, Q). We see that keeping the quarantine is the most advantageous choice that can be selected by the players for Italian people. However, we notice that Italy's payoff of the quarantine is worse than the payoff of South Korea, $(0.9840, 0.9840)$. Therefore, we can say that the first stage in Italy is more critical than South Korea. Moreover, it can be seen that the loss for (Q', Q') strategy of Italy is 7 times more than the loss of South Korea, which implies that the pandemic in Italy started worse than South Korea.

As the second stage, we analyze the spread case. At this stage, the possibility of getting the virus is 0.1849 and not getting the virus is 0.8151, which are evaluated by Eq. (1). The payoff bimatrix is generated by the procedure as,

$$It_{spread} = \begin{bmatrix} & Q & Q' \\ Q & (0.8151, 0.8151) & (0.8151, -0.1849) \\ Q' & (-0.1849, 0.8151) & (-0.3698, -0.3698) \end{bmatrix}$$

The Nash equilibrium is also (Q, Q) strategy with the payoffs $(0.8151, 0.8151)$. It is clear that the payoffs in the second stage is decreased comparing to the first stage. The loss of (Q', Q') strategy, i.e. breaking the quarantine, is increased more than 50%. Thus, we may say that breaking the quarantine when the spread rate is high becomes 50% more risky.

In addition to these analysis, we investigate the results if the players have to make selection for a second time during the spread stage. In other words, we convert the game to a repeated game same as it is done in the analysis of South Korea. We create the second round payoff bimatrix $It_{Round_{II}}$ for Italy by using It_{spread} bimatrix as,

$$It_{Round\_II} = \begin{bmatrix} (\overline{1.6302}, \widehat{1.6302}) & (\overline{1.6302}, 0.6302) & (\overline{1.6302}, 0.6302) & (\overline{1.6302}, -0.3698) \\ (0.6302, \widehat{1.6302}) & (0.6302, 0.6302) & (0.6302, 0.6302) & (0.6302, 0.3698) \\ (0.6302, \widehat{1.6302}) & (0.6302, 0.6302) & (0.4453, 0.4453) & (0.4453, -0.5547) \\ (-0.3698, \widehat{1.6302}) & (-0.3698, 0.6302) & (-0.5547, 0.6302) & (-0.7369, -0.7396) \end{bmatrix}$$

We obtain the Nash equilibrium point as $(1.6302, 1.6302)$, that is $(QQQQ)$ strategy, if we find the equilibrium points as we did for the previous illustrations. On the other hand, it is clearly seen that the worst strategy is $(-0.7369, -0.7396)$ which is $(Q'Q'Q'Q')$. If we compare the results with South Korea's results, we see that breaking the quarantine is 5 times more dangerous when the rate of the virus spread is high. Therefore, we may say that the Covid-19 crisis in Italy is worse than S. Korea.

3.3. Case III: Turkey

In this analysis, we consider Turkey's data set starting from the date the very first Covid-19 diagnosis, 11.03.2020, and the last data declared by the ministry of health in 03.05.2020 [45]. We follow up the same procedure to analyze the data as we did in the previous cases. We refer to the days between 11.03.2020 and 25.03.2020 as the start and the time period between 26.03.2020–03.05.2020 is named as the spread for Turkey. The payoff matrix generated by using the average of the tests/diagnoses calculated by Eq. (1) for the first stage is below, and then we put a mark on the binaries to find Nash equilibrium point in the payoff matrix,

$$Tur_{start} = \begin{bmatrix} & Q & Q' \\ Q & (\overline{0.9502}, \widehat{0.9502}) & (\overline{0.9502}, -0.0498) \\ Q' & (-0.0498, \widehat{0.9502}) & (-0.0996, -0.0996) \end{bmatrix}$$

The Nash equilibrium is the same as the previous examples which is $(0.9502, 0.9502)$ or equivalently (Q, Q) strategy. It is clearly better than the same stage of Italy and slightly worse than South Korea. However, the comparison of the worst strategies, that is (Q', Q') strategy, shows that Turkey is better than Italy and worse than South Korea. As the second stage, we obtain the following payoff matrix for the spread stage,

$$Tur_{spread} = \begin{bmatrix} & Q & Q' \\ Q & (\overline{0.8735}, \widehat{0.8735}) & (\overline{0.8735}, -0.1265) \\ Q' & (-0.1265, \widehat{0.8735}) & (-0.2530, -0.2530) \end{bmatrix}$$

The equilibrium point is $(0.8735, 0.8735)$ which is the same strategy as in the previous cases. Even though it is worse than South Korea's payoff, it is better than Italy's payoff. Therefore, we may conclude that the pandemic in Turkey expands slower than Italy and faster than South Korea.

As a final analysis, we generate a repeated game, as it is done in Case I and Case II above, for Turkey by using Tur_{spread} data set. Then, we have the payoff matrix of the repeated game as below,

$$Tur_{Round\_II} = \begin{bmatrix} (\overline{1.7470}, \widehat{1.7470}) & (\overline{1.7470}, 0.7470) & (\overline{1.7470}, 0.7470) & (\overline{1.7470}, -0.2530) \\ (0.7470, \widehat{1.7470}) & (0.7470, 0.7470) & (0.7470, 0.7470) & (0.6205, -0.3795) \\ (0.7470, \widehat{1.7470}) & (0.7470, 0.7470) & (0.6205, 0.6205) & (0.6205, -0.3795) \\ (-0.3795, \widehat{1.7470}) & (-0.3795, 0.6205) & (-0.3795, 0.6205) & (-0.5060, -0.5060) \end{bmatrix}$$

As in the start case, we find the pure Nash equilibrium point as $(1.7470, 1.7470)$. We can point out that the result is better than Italy and worse than South Korea.

In addition to above analyses, the importance of the quarantine in the future of the pandemic for all countries can be modeled and analyzed with the same approach.

4. Conclusion and discussion

4.1. Conclusion

We investigate the observance of the quarantine rules during the first wave of the Covid-19 pandemic in three different countries by using a game-theoretical approach. In order to do this, we first select three countries as examples: South Korea, Italy, and Turkey, for the analysis. First, we divide the process of the pandemic into three-stage as the start, the spread, the end according to the data provided by officials. Then, we obtain corresponding bimatices for each country by taking into account the daily diagnosis numbers/number of tests for each stage under the procedure as it is described in chapter 2. Later on, we reconsider each country in the view of the repeated game approach to present the quarantine effects more explicitly. According to the analyses are being done, we obtain the following results:

In the first stage of the spread, the best payoff belongs to South Korea with 0.9840. If we compare the first stages of Italy and Turkey since we have almost the same number of data for each country, Turkey has a higher payoff with 0.9205 comparing to Italy, 0.8858. It is clear that (Q, Q) strategy, in other words, staying the quarantine is the best action for all countries in the first stage of the pandemic. However, it is important to notice that (Q', Q') strategies shows the level of seriousness of the pandemic in the countries. We see that Italy's payoff, -0.2284 is the worst among the others. It is 2 times worse than Turkey's payoff, -0.0996, and more than 7 times worse than South Korea's payoff -0.0320. It implies that the pandemic started very fast in Italy comparing to the other two countries.

In the spread case, in other words the second stage, the payoffs of all countries in our study decrease due to the increasing number of cases. We may conclude that the risk of infection gets higher during the second stage. However, South Korea

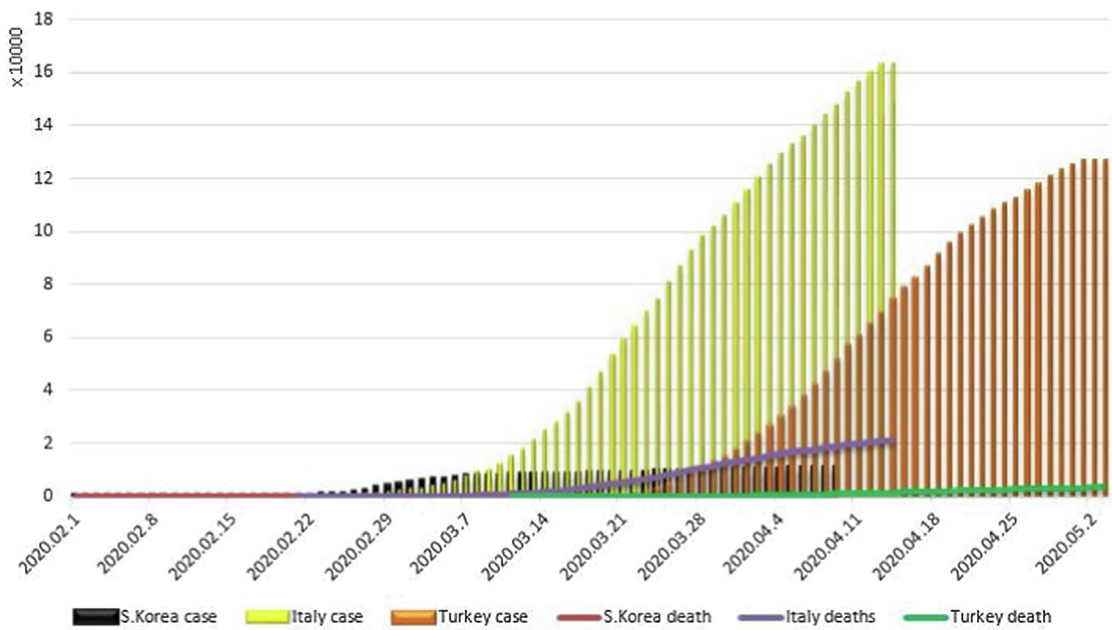

Fig. 2 The Cumulative Deaths and Cases of S.Korea, Italy and Turkey.

still has the best payoff with 0.9634. Turkey follows South Korea with 0.8735. Italy's payoff is still the worst with 0.8151. The best action for all countries is (Q,Q) as in the first stage. If we compare payoffs of (Q',Q') strategy, we see that Italy has the worst payoff -0.3698, which is 5 times worse than South Korea, -0.0732, and 1.5 times worse than Turkey, -0.2536.

In the third case which is investigated only for South Korea since they take control over the spread in the investigated time period, we see that the payoff of (Q,Q) strategy increased, 0.9870, it is even slightly greater than the payoff in the first stage, 0.9840. This increment may be the result of the control over the pandemic.

On the other hand, we study the case when any player disobeys the precautions during the second stage which is the level of the spread is maximum for these three countries. The Nash equilibrium points did not change, it is still (Q,Q,Q,Q). In other words, if one has to make a choice between staying in the quarantine or breaking the quarantine, it is always better to select to keep the quarantine, that is, the action of Q. The player's payoffs for each country doubles in the choice of (Q,Q) strategy once again, $1.9268, 1.6302, 1.7470$ for South Korea, Italy, and Turkey respectively. However, it is crucial to realize that the losses in the choice of the breaking the quarantine strategy, for these three countries increases, which means their risk of infection gets higher. Italy has the worst payoff with -0.7369 comparing to South Korea and Turkey. South Korea has the best payoff -0.1469 and Turkey follows South Korea with -0.5060.

In order to see the situation from a different perspective, we calculate the death rates over the diagnosed cases during the given date intervals for each country with $\lambda_{CountryName} = \frac{TotalNumberofDeaths}{TotaNumberofDiagnosedCase}$, we get the results as $\lambda_{SouthKorea} = 0.0196, \lambda_{Italy} = 0.1296$ and $\lambda_{Turkey} = 0.0270$. These results can be ordered as $\lambda_{SouthKorea} \leqslant \lambda_{Turkey} \leqslant \lambda_{Italy}$. Although the rates of death also depend on the other parameters except for the quarantine effects (i.e. number of hospitals, doctors, intensive care, in short, health care system, etc), the order confirms the results for the quarantine requirement of each country obtained by game theory approach. However, the cumulative deaths and diagnosis of the countries are presented in Fig. 2 (see Appendix A). In this figure, the death curve of Italy has a significant difference comparing to South Korea's and Turkey's. On the other hand, even though the total number of diagnoses in Italy and Turkey seems similar, the number of deaths in Turkey is very small compared with Italy. The best country among these three countries is clearly South Korea since the number of deaths and cases are extremely small and barely seen in Fig. 2.

4.2. Discussion

In order to see the pandemic situation from a broad perspective, we present some data about the pandemic belonging to the developed and developing countries in Table 1 (see Appendix B). Even though Italy is a developed country, the situation in Italy seems to be worse than most of the developed and developing countries in the table. This result may be happened due to taking the precautions late since a day makes a great difference in the spread during pandemics. It is caused by the corruption of the health system in Italy. However, Russia, which is a developing country, stands in the first line of the Table 1.

On the other hand, the table shows that the developing countries hold the very first places in the table comparing to the developed countries except for South Korea. Therefore, we may conclude that precautions (self or general quarantine), healthcare network, and capacity for the leading countries in the table are at least as good and enough in comparison with the others.

Appendix A. We present the number of deaths and cases for South Korea, Italy, and Turkey in Fig. 2. We see that the number of cases in S. Korea is extremely small compared to Italy and Turkey. Additionally, we barely notice the death curve of S.Korea since the number is very small. On the other hand, the comparison of the death curves of Italy and Turkey demonstrates that the control over the disease is better in Turkey since its death curve is quite under the curve of Italy even though the numbers of the cases in both countries are similar.

Appendix B. The total number of diagnoses, deaths, and rate of deaths of some developed and developing countries are shown in Table 1. We can see that most of the developed countries are placed in the last lines of the table except S. Korea. The first places belong to developing countries which is an unexpected situation.

Table 1 The Data for some Developed vs Developing Countries on 03 May 2020.

Countries	# of Total Diagnosis	# of Total Deaths	The rate of Deaths/ Diagnosis
Russia	134687	1280	0.0095
Pakistan	20084	4576	0.0228
South Korea	10793	252	0.0233
Turkey	126045	3397	0.0270
India	42505	1391	0.0327
Germany	167007	6866	0.0411
U.S.A.	1188112	68597	0.0577
Canada	59474	3682	0.0619
Iran	97424	6203	0.0638
Brazil	101147	7025	0.0695
Italy	210717	28884	0.1371
U.K.	186599	28446	0.1370

References

[1] M. Shubik, Game Theory and Related Approaches to Social Behaviour: Selections, John Wiley & Sons, 1964.

[2] K.G. Guseinov, E. Akyar, S.A. Duzce, Oyun Teorisi Catisma ve Anlasmanin Matematiksel Modelleri, Seckin (in Turkish), 2010.

[3] O.G. Haywood Jr., Military decision and game theory, J. Oper. Res. Soc. Am. 2 (4) (1954).

[4] J. Von Neumann, O. Morgenstern, Theory of Games and Economic Behaviour, Princeton University Press, 1944.

[5] K. Borch, Application of game theory to some problems in automobile insurance, ASTIN Bull.: J. IAA 2 (2) (1962) 208–221.

[6] R.C. Agrawal, E.O. Heady, Application of game theory models in agriculture, J. Agric. Econ. 19 (2) (1968) 207–218.

[7] G.H. Snyder, 1971. Prisoner's dilemma and Chicken models in international politics, Int. Stud. Quart., Nol. 15, No.1 (Mar., 1971), 66–103.

[8] M. Lee, R.D. McKelvey, H. Rosenthal, Game theory and the French apparentements of 1951, Int. J. Game Theory 8 (1979) 27–53.

[9] C.S. Fisk, Game theory and transportation systems modelling, Transport. Res. Part B: Methodol. 18 (4–5) (1984) 301–313.

[10] D.W.K. Yeung, A differential game model of a market of substitutable products, Eur. J. Oper. Res. 90 (3) (1996) 599–608.

[11] R. Aumann, T. Schelling, Contributions to Game Theory: Analysis of Conflict and Cooperation, Nobel Prize Econ. Documents (2005), 2005–1.

[12] M. Doebeli, C. Hauert, Models of cooperation based on the Prisoner's Dilemma and the Snowdrift game, Ecol. Lett. 8 (2005) 748–766.

[13] K. Madani, Game theory and water resources, J. Hydrol. 381 (3–4) (2010) 225–238.

[14] K. Chlebik, Terrorism and game theory: From the terrorists' point of view, Pepperdine, Policy Rev. 3 (2010) Article 3.

[15] R. Young, Transition costs in secessions, with a brief application to Scotland, Oxford Review of Economic PolicyVolume 30, Issue 2, September 2014, Article number gru015, 392–405, 2014.

[16] Y. Köse, Küresel finansal yaptırımlar: Oyun teorisi yaklaşımı ile ampirik bir uygulama, Maliye ve Finans Yazıları 1 (103) (2014) 10–20.

[17] G. Diesen, EU and NATO relations with Russia: After the collapse of the Soviet Union, Routledge, 2015.

[18] B. Erkut, A hung parliament in the north: Outside options after the 2003 election en route to the Annan Plan referendum of 2004, Cyprus Rev. 27 (2) (2015) 105–122.

[19] F. Farhidi, K. Madan, A Game Theoretic Analysis of the Conflict over Iran's Nuclear Program, Proceedings - 2015 IEEE International Conference on Systems, Man, and Cybernetics, SMC 201512 January 2016, Article number 7379250, 617–622, 2015.

[20] A.T. Little, 2017, Are non-competitive elections good for citizens?, J Theoret. Polit. 29 (2) (2017) 214–242.

[21] A.B. Caleiro, Global development and climate change: A game theory approach, Climate Change Global Develop. (2019) 17–35.

[22] J. Moldalieva, J. Heathershaw, Playing the "Game" of transparency and accountability: Non-elite politics in Kyrgyzstan's natural resource governance, Post-Soviet Affairs 36 (2) (2020) 171–187.

[23] B. İzgi, M. Özkaya, A New Perspective to the Solution and Creation of Zero-Sum Matrix Game with Matrix Norms, Appl. Math. Comput. 341 (2019) 148–159.

[24] B. İzgi, M. Özkaya, The demonstration of the necessity of agriculture insurance by the game theory: Matrix norm approach, Afyon Kocatepe University J. Sci. Eng. 20 (5) (2020) 824–831 (in Turkish).

[25] B. Ivorra, M.R. Ferrandez, M. Vela-Perez, A.M. Ramos, Mathematical modeling of the spread of the coronavirus disease 2019 (COVID-19) taking into account the undetected infections. The case of China., ResearchGate Preprint, 1 April 2020, 2020.

[26] A.J. Kucharski, T.W. Russel, C. Diamond, Yang Liu, J. Edmunds, Sebastian Funk, R.M. Eggo, Early dynamics of transmission and control of COVID-19: a mathematical modelling study, Lancet Infect Dis (2020), published online March 11, 2020.

[27] J. Hellewell, S. Abbott, A. Gimma, I.N. Bosse, C.I. Jarvis, T.W. Russel, J. Munday, A.J. Kucharski, J. Edmunds, 2020, Feasibility of controlling COVID-19 outbreaks by isolation of cases and contacts, The Lancet Global Health 8 (4) (2020) e488–e496.

[28] Z. Hu, Q. Ge, S. Li, L. Jin, M. Xiong, 2020, Artificial intelligence forecasting of Covid-19 in China, arXiv:2002.07112.

[29] Z. Cakir, H.B. Savas, A mathematical modelling for the covid-19 pandemic in Iran, Ortadogu Tıp Derg 12 (2) (2020) 206–210.

[30] N. Wang, Y. Fu, H. Zhang, H. Shi, An evaluation of mathematical models for the outbreak of Covid-19, Precis. Clin. Med. 3 (2) (2020) 85–93.

[31] A. Caparros, M. Finus, The corona-pandemic: A game theoretical perspective on regional and global governance, Environ. Resource Econ. 76 (2020) 913–927.

[32] A. Atagana, S.İ. Araz, 2020, Mathematical model of Covid-19 spread in Turkey and South Africa: theory, methods, and applications, Adv. Diff. Eqs. 659 (2020).
[33] S.İ. Araz, Analysis of a Covid-19 model: Optimal control, stability and simulations, Alexandria Eng. J. 60 (1) (2021) 647–658.
[34] M. Perc, N.G. Miksic, M. Slavinec, A. Stozer, Forecasting COVID-19, Front. Phys. 8 (2020) 127.
[35] S. Momtazmanesh, H.D. Ochs, L.Q. Uddin, M. Perc, J.M. Routes, D.N. Vieira, W. Al-Herz, S. Baris, C. Prando, L. Rosivall, A.H. Abdul Latiff, T. Ulrichs, V. Roudenok, J.C. Aldave Becerra, D.B. Salunke, E. Goudouris, A. Condino-Neto, A. Stashchak, O. Kryvenko, N. Rezaei, All together to fight novel coronavirus disease (COVID-19), Am. J. Trop. Med. Hygiene 102 (2020) 1181–1183.
[36] M.G. Hancean, M. Perc, J. Lerner, 2020 Early spread of Covid-19 in Romania: Important cases from Italy and human-to-human transmission networks, R. Soc. Open Sci. 7 (2020) 200780.
[37] A. Atagana, S.İ. Araz, Nonlinear equations with global differential and integral operators: Existence, uniqueness with application to epidemiology, Results Phys., Vol 21, 103593, 2021.
[38] T.S. Ferguson, 2014, Game Theory, Mathematics Department UCLA, Part III, 2.Edition.
[39] E. Prisner, Game Theory Through Examples, The Mathematical Association of America, 2014.
[40] https://ourworldindata.org/covid-testing, (Last access: 08.01.2021).
[41] https://coronavirus.jhu.edu/map.html, (Last access:08.01.2021).
[42] P.D. Straffin, Game Theory and Strategy, The Mathematical Association of America, 1993.
[43] N.E. Barron, Game Theory: An Introduction, John Wiles & Sons Inc., 2013.
[44] https://lab.gedidigital.it/gedi-visual/2020/coronavirus-i-contagi-in-italia/, (Last access:08.01.2021).
[45] https://covid19.saglik.gov.tr/, (Last access: 08.01.2021).
[46] https://www.aa.com.tr/en/latest-on-coronavirus-outbreak/who-official-praises-turkey-s-fight-against-coronavirus/1799351, (Last access: 28.11.2020).