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Standard stellar luminosities: what are typical and limiting accuracies in the era after *Gaia*?

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ABSTRACT
Methods of obtaining stellar luminosities (L) have been revised and a new concept, standard stellar luminosity, has been defined. In this paper, we study three methods: (i) a direct method from radii and effective temperatures; (ii) a method using a mass–luminosity relation (MLR); and (iii) a method requiring a bolometric correction. If the unique bolometric correction (BC) of a star extracted from a flux ratio (f_V/f_{Bol}) obtained from the observed spectrum with sufficient spectral coverage and resolution are used, the third method is estimated to provide an uncertainty ($\Delta L/L$) typically at a low percentage, which could be as accurate as 1 per cent, perhaps more. The typical and limiting uncertainties of the predicted L of the three methods were compared. The secondary methods, which require either a pre-determined non-unique BC or MLR, were found to provide less accurate luminosities than the direct method, which could provide stellar luminosities with a typical accuracy of 8.2–12.2 per cent while its estimated limiting accuracy is 2.5 per cent.

Key words: stars: fundamental parameters – stars: general.

1 INTRODUCTION

The luminosity of a star is not a directly observable parameter. Rather, it is an empirical parameter to be computed from observable parameters: a radius (R) and an effective temperature (T_{eff}). Therefore, the most reliable stellar luminosities so far are the ones that are calculated directly from the Stefan–Boltzmann law, $L = 4\pi R^2 \sigma T_{eff}^4$, using the observed radii and effective temperatures of detached double-lined eclipsing binaries (DDEBs; Andersen 1991; Torres, Andersen & Giménez 2010; Eker et al. 2014, 2015, 2018, 2020). These parameters are obtained from the simultaneous solutions of the radial velocity and eclipsing light curves of DDEBs and/or analysis of disentangled spectra of their components (Hadrava 1995; Bakış et al. 2007). Except for a very limited number of nearby single stars, which have radii available by interferometry and any kind of observed effective temperatures, the direct method, alas, has a serious defect. This method is not applicable to single stars and visual and spectroscopic binaries (or multiple systems) because their stellar radii are not observable directly. Moreover, effective temperatures implied by intrinsic colours, in most cases, are also not observable because of interstellar reddening. The direct method even faces difficulties for contact, semidetached and even close binaries because of proximity effects, which deform star shapes and thus the Stefan–Boltzmann law is not applicable directly.

In addition to this limited primary source of stellar luminosities (the direct method), there are two other secondary sources, which indirectly provide stellar luminosities. These are indirect because both rely upon a prior relation determined by the most accurate stellar parameters, mostly from DDEBs.

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2 TWO SECONDARY METHODS

2.1 Mass–luminosity relations for predicting L

From a historical point of view, the first of the secondary sources is the main-sequence mass–luminosity relation (MLR), $L \propto M^{\alpha}$. Despite being applicable only to main-sequence stars, this method has the power to increase the availability of stellar luminosities in the case of single stars (if their masses are known or estimated somehow) and visual binaries (or multiple systems) with orbital parameters. If orbital parameters could be extracted from a visual orbit, then component masses would be known according to Kepler's third law. Note that, unlike a visual binary with an orbital inclination deduced from its visual orbit, a spectroscopic binary is without an orbital inclination because it is not possible to deduce the orbital inclination from the observed spectra. Therefore, spectroscopic binaries do not provide the true masses of components, except for a mass ratio or a mass function. Hence, this method is not applicable to spectroscopic binaries unless orbital inclinations are somehow available.

The main-sequence MLR was discovered independently by Hertzsprung (1923) and Russell, Adams & Joy (1923) empirically in the middle of the first half of the 20th century. As newer and more accurate data came along, it has been revised, updated and improved many times (Eddington 1926; McLaughlin 1927; Kuiper 1938a; Petrie 1950a,b; Strand & Hall 1954; Eggen 1956; McCluskey & Kondo 1972; Cester, Ferluga & Boehm 1983; Griffiths, Hicks &

Milone 1988; Demircan & Kahraman 1991). Early relations were demonstrated as mass–absolute bolometric magnitude diagrams, some with the best-fitting curve and some without. First, Eggen (1956) attempted to define the power of mass (alpha) so the relation is expressed as $L = \mu^{3.1}$, where μ is defined as $\mu = a^3/P^2\varpi^3$ from Kepler's harmonic law in which a and ϖ are the semimajor axis and parallax of the double star in units of arcsec, P is orbital period in yr and μ is the mass of the system in solar masses. Meanwhile, McCluskey & Kondo (1972) tried to establish the relation as $M = \alpha L^{\beta}$, where M and L are the masses and luminosities of components in which α and β are the constants determined on various mass–absolute bolometric magnitude diagrams. However, Cester et al. (1983), Griffiths et al. (1988) and Demircan & Kahraman (1991) preferred to study a mass–luminosity diagram in order to define unknown constants on the classical form of the MLR ($L \propto M^{\alpha}$). This was done either by fitting a curve to all data or by dividing the mass range into two (low-mass and high-mass stars) or three (high-mass, intermediate-mass or solar low-mass stars) in order to define the inclination of the linear MLR (power of M) and its zero-point constant on $\log M - \log L$ diagrams.

Andersen (1991) objected to defining any form of MLR and preferred to display the $\log M - \log L$ diagram without a curve fit, because the scatter from the curve is not only due to observational errors but also due to abundance and evolutionary effects. He claimed that '[...] departures from a unique relation are real'. So, if there is no unique function to represent data, why bother to define one? Because of this objection, Henry (2004) and Torres et al. (2010) also displayed their diagrams without a curve. Some authors, such as Gafeira, Patacas & Fernandes (2012), Torres et al. (2010), Benedict et al. (2016), Moya et al. (2018) and, recently, Fernandes, Gafeira & Andersen (2021), have deviated from the tradition of defining the MLR, which can be used in both directions (L computed from M or M computed from L), with the idea that the luminosity of solar-type single stars can be obtained from observations with fair accuracy but not the mass (Fernandes et al. 2021). There are exceptions: Gorda & Svechnikov (1998) preferred the form $M_{\rm Bol} = a + b\log M$, where $M_{\rm Bol}$ is the absolute bolometric magnitude and M is the mass; Henry & McCarthy (1993) preferred $\log M = aM_{\xi} + b$ for infrared colours, where M_{ξ} denotes the absolute magnitude at the J, H and K bands, and $\log M = aM_V^2 + bM_V + c$ is used for the V band to express various MLRs, with unknown coefficients a, b and c to be determined by data on the diagram; and Malkov (2007) defined MLR and inverse MLR functions. Therefore, the MLR should be established only for estimating the mass of single stars from other astrophysical stellar parameters such as its luminosity, metallicity (Z) and age, not for estimating luminosities from masses (Fernandes et al. 2021). Nevertheless, the predicted relation is still called the MLR despite the fact that it is not a relation solely between mass and luminosity but also includes metallicity and age as observable parameters. Devised for estimating mass rather than luminosity, the MLR of this kind is not suitable for this study.

The classical MLR in the form $L \propto M^{\alpha}$ was appreciated by Ibanoğlu et al. (2006) when they were comparing MLRs for detached and semidetached Algols. The tradition of the MLR in the form $L \propto M^{\alpha}$ (or reducible to it) has been continued by Eker et al. (2015, 2018). Note that any curve or a polynomial of any degree fitting data on a $\log M - \log L$ diagram is reducible to the form $L \propto M^{\alpha}$ because the derivative of the fitting function at a given mass gives the value of alpha. The advantage of such an MLR is not only that it works both ways (L from M, or M from L), but also that it permits one to relate typical masses and luminosities of main-sequence stars in general. Because in this study we are primarily interested in estimating typical accuracies of mass and luminosities, the six-piece classical MLRs of Eker et al. (2018), as the most recent determined MLRs, are more suitable for this study than any of the other MLR forms.

Although L obtained from a classical MLR would be a unique value for a given mass (M), it is akin to the mean value of all luminosities from the zero age main-sequence (ZAMS) to the terminal age main-sequence (TAMS) of stars with the same mass but with different ages and various chemical compositions. Thus, the uncertainty of obtained L is expected to be very large for those who are looking for an accurate L of a star in question. Still, this was the only method for producing the luminosities of single stars with known masses and of visual binary or multiple systems with visual orbits in earlier times, when the third source using bolometric corrections (BC) was not available yet or the BC values were not as accurate as today.

2.2 Bolometric corrections for predicting *L*

Perhaps the most powerful secondary method for obtaining L is the method using bolometric correction *(BC)*. It appears to be even more powerful than the direct method not only because it is applicable to all stars, single or binary (or multiple), but also because it is more practical and easier to use. The method can even work with a single observation at a preferred filter, let us say V filter. Nowadays, it could be accurate at a millimag level, perhaps more accurate, if the star is bright enough, though not a binary or a multiple system. For binaries and multiple systems, however, the method requires the relative light contributions of the components. The only disadvantage, compared with the direct method, is that a trigonometric parallax (ϖ) and reddening (or extinction) of the stars must be provided. Nevertheless, despite these advantages, the method is still secondary because it does not work if a pre-determined BC value is not available. These values can be obtained from any of the tabulated BC tables in the literature, or from the $BC - T_{\rm eff}$ and $BC - M/{\rm M}_{\odot}$ relations (Flower 1996; Eker et al. 2020, 2021), or similar relations if available.

Analytical $BC - T_{\rm eff}$ relations, however, have been determined only by Flower (1996) and Eker et al. (2020). The rest of the available *BCs* are all in tabular format (Kuiper 1938b; Popper 1959; McDonald & Underhill 1952; Johnson 1964, 1966; Heintze 1973; Code et al. 1976; Malagnini et al. 1985; Cayrel et al. 1997; Bessell, Castelli & Plez 1998; Girardi et al. 2008; Sung et al. 2013; Chen et al. 2019), where the variation of BC values with other stellar parameters, such as spectral type, intrinsic colour, luminosity class, metallicity and surface gravity, may also be given.

It was Torres (2010) who first noticed inconsistencies in the use of BC_V values that may lead to errors of up to 10 per cent or more in the derived luminosity equivalent and about 0.1 mag or more uncertainty in the bolometric magnitudes. According to Torres (2010), the problems arise from the arbitrariness attributed to the zero-point of the BC scale. Recently, Eker et al. (2021) revised the zero-points of the BC scales on the tabulated tables and confirmed Torres (2010) independently. According to the results of Eker et al. (2021), there could be up to 0.1-mag systematic shifts of BC values corresponding to systematic errors of up to 10 per cent in predicted stellar luminosities, which are intolerable in the era after *Gaia*.

In this paper, we must first re-emphasize the IAU 2015 General Assembly Resolution B2, which Eker et al. (2021) relied upon in solving the problems originating from the arbitrariness attributed to the BC scale. Fixing the zero-point of the BC according to the IAU 2015 General Assembly Resolution would actually mean the standardization of BC. Briefly, the standardization of BC values was a solution suggested by Eker et al. (2021) to remove uncertainties on both stellar absolute bolometric magnitudes and predicted stellar luminosities caused by the arbitrariness attributed to the zero-point constants of *BC* values appearing in the literature.

Because stellar luminosities are used by astronomers to estimate Galactic and extragalactic structures and luminosities, which can then be used to estimate Galactic and extragalactic distances as well as the luminous mass contained in galaxies and the Universe, the standardization apparently is expected to have a widespread effect on dark matter research, Hubble law and cosmological models. For this reason, it is necessary to be careful about uncertainties and errors reflected in *BC* values, which naturally propagate to stellar luminosities. Not only are the luminosities of stars affected, but also deep space and cosmology research. In this paper, we intend to explain how these negative propagating effects could be minimized and how to estimate and compare accuracies of the stellar luminosities predicted by the three methods. We also discuss the need to emphasize the differences between standard and non-standard luminosities.

3 DATA

The preliminary data for this study are taken from the Catalogue of Stellar Parameters from the Detached Double-Lined Eclipsing Binaries in the Milky Way (Eker et al. 2014), which contains 514 stars (257 systems). Although the updated catalogue (Eker et al. 2018) was expanded to include 639 stars (318 binaries and one eclipsing spectroscopic triple), after removing stars with errors greater than 15 per cent on both mass and radius, and eliminating stars belonging to globular clusters, then finally choosing main-sequence stars with a mass of $0.179 \le M/\mathrm{M}_{\odot} \le 31$ and metal abundance $0.008 \le Z \le 0.040$ ranges within the limits of theoretical ZAMS and TAMS according to PARSEC models (Bressan et al. 2012), 509 stars were retained by Eker et al. (2018) to study the MLR of the sample representing the nearby stars in the Galactic disc in the solar neighbourhood.

The most accurate stellar luminosities and propagated uncertainties were calculated according to the direct method (method 1) using the most accurate radii and effective temperatures and the associated observed uncertainties of the 509 main-sequence stars, which are the 'components' of DDEBs in the updated catalogue. In this study, we use the six-piece MLRs in the form of $\log L = a \log M + b$ calibrated by the data on the $\log M - \log L$ diagram covering the mass range $0.179 \le M/\mathrm{M}_{\odot} \le 31$ and metal abundance range $0.008 \le Z \le 0.040$ for the main-sequence stars in the Galactic disc in the solar neighborhood by Eker et al. (2018).

4 RELATIVE ACCURACIES ACCORDING TO THE THREE METHODS

4.1 Relative uncertainty of *L* according to method 1

The uncertainty of the luminosity by the direct method using the Stefan–Boltzmann law can be calculated by

$$\frac{\Delta L}{L} = \sqrt{\left(2\frac{\Delta R}{R}\right)^2 + \left(4\frac{\Delta T_{\mathrm{eff}}}{T_{\mathrm{eff}}}\right)^2}. \quad (1)$$

where $\Delta R/R$ and $\Delta T_{\mathrm{eff}}/T_{\mathrm{eff}}$ are relative observational random errors of a star's radius and effective temperature. These usually come from simultaneous solutions of the light and radial velocity curves of DDEBs and spectral analysis of the disentangling spectra of the system's components.

4.2 Relative uncertainty of *L* according to method 2

In order to estimate relative uncertainty of *L* using the method of error propagation, the form of the adopted MLR ($L \propto M^{\alpha}$) indicates

$$\frac{\Delta L}{L} = \alpha \frac{\Delta M}{M}, \quad (2)$$

where $\Delta M/M$ is the relative uncertainty of the observed mass of a star, α is the power of *M* and $\Delta L/L$ is the relative uncertainty of the predicted luminosity. According to Eker et al. (2015, 2018), equation (2) is invalid because the dispersions on the $\log M - \log L$ diagram are not only due to observational uncertainties of *M*, but also due to the differences in the ages and chemical compositions of the stars in the sample (Andersen 1991; Torres et al. 2010; Eker et al. 2015, 2018). It is better to use

$$\frac{\Delta L}{L} = \frac{SD}{0.4343}, \quad (3)$$

where *SD* is the standard deviation of data from the MLR function, which should be chosen according to the mass of the star in question. There are six MLR functions with standard deviations and inclinations, already computed by Eker et al. (2018), which are used in this study. Only if the cases are

$$\alpha \frac{\Delta M}{M} > \frac{SD}{0.4343}$$

would equation (2) then be valid. Because the typical relative uncertainties of *M* are in the order of 1–2 per cent (Eker et al. 2014), equation (2) is not valid. It would be valid for stars with relative uncertainties bigger than about 6 per cent for low-mass ($M < 2.4\,\mathrm{M}_{\odot}$) stars, and relative uncertainties bigger than about 10 per cent for high-mass ($M > 2.4\,\mathrm{M}_{\odot}$) stars (Eker et al. 2015).

4.3 Relative uncertainty of *L* according to method 3

The same data set of 509 main-sequence stars used for method 2 is also used for method 3 here, which uses a pre-determined bolometric correction (*BC*) for predicting the luminosity of a star. Unfortunately, many of the binary systems containing the 509 stars as components of DDEBs had to be eliminated because some of the binaries have not been observed in standard *V* magnitudes, or the light contributions of components in the *V* band could not be achieved, or a reliable trigonometric parallax for the star (ϖ) did not exist, or reliable interstellar reddening could not be found.

Eker et al. (2020) could find only 206 binaries that have at least one component in the main sequence (194 systems with components, eight systems with primaries, four systems with secondaries on the main sequence). This leaves a total of 400 main-sequence stars that are eligible to compute bolometric correction (*BC*) coefficients in the *V* band. A standard $BC_{\mathrm{V}} - T_{\mathrm{eff}}$ curve (a fourth-degree polynomial), which is valid in the range $3100 \le T_{\mathrm{eff}} \le 36\,000$ K, was calibrated. The coefficients of the polynomial, errors of the coefficients, standard deviation ($SD = 0.215$) and correlation coefficient ($R^2 = 0.941$) have already been announced by Eker et al. (2020). Table 5 of Eker et al. (2020), which contains the necessary statistics, was adopted for this study as the basic data to compute the luminosity of a star (*L*) according to method 3.

In this method, the *L* of a star could be achieved according to the following relation,

$$M_{\mathrm{Bol}} = -2.5 \log L + C_{\mathrm{Bol}}, \quad (4)$$

where M_{Bol} is the absolute bolometric magnitude of the star and $C_{\mathrm{Bol}} = 71.197\,425\ldots$ if *L* is in SI units, and/or $C_{\mathrm{Bol}} =$

88.697 425… if L is in cgs units (see IAU 2015 General Assembly Resolution B2 and Eker et al. 2021). The only requirement is that the $M_{\rm Bol}$ of the star must be known beforehand so that L can be extracted. $M_{\rm Bol}$ is available according to following relation,

$$M_{\rm Bol} = M_{\rm Filter} + BC_{\rm Filter} = M_{\rm V} + BC_{\rm V}, \tag{5}$$

where $M_{\rm V}$ and $BC_{\rm V}$ are the absolute visual magnitude and associated bolometric correction for the star in question. Notice that the absolute bolometric magnitude of a star is independent of the filter used in observations. Because V filter observations are the oldest and most available, the V filter was chosen here to symbolize all BC values of many photometric bands. If the $BC_{\rm V}$ value of the star is available, then an additional step must be taken to obtain $M_{\rm V}$ directly from observable parameters according to the following relation:

$$M_{\rm V} = V + 5\log\varpi + 5 - A_{\rm V}. \tag{6}$$

Here, V is the apparent visual magnitude, ϖ is the trigonometric parallax of the star, which is nowadays available up to 21 mag (Brown et al. 2021), and $A_{\rm V}$ is the extinction in the V band, which can be ignored if the star is in the Local Bubble (Leroy 1993; Lallement et al. 2019) or it can be estimated using galactic dust maps (e.g. Schlafly & Finkbeiner 2011; Green et al. 2019).

It is clear in this method that the only uncertainty to propagate up to L comes from $M_{\rm Bol}$. It can be considered that a well-defined constant (see IAU 2015 General Assembly Resolution B2) $C_{\rm Bol}$ does not make any contribution, and thus

$$\frac{\Delta L}{L} = \frac{\Delta M_{\rm Bol}}{2.5\log e} = 0.921 \times \Delta M_{\rm Bol}. \tag{7}$$

However, equation (5) indicates

$$\Delta M_{\rm Bol} = \sqrt{\Delta M_{\rm V}^2 + \Delta BC_{\rm V}^2 + ZPE_{\rm V}^2}. \tag{8}$$

which means that there are three possible error contributions to $\Delta M_{\rm Bol}$. These are: (i) random observational errors associated with the absolute visual magnitude ($\Delta M_{\rm V}$); (ii) the error of the BC value itself ($\Delta BC_{\rm V}$); (iii) the zero-point uncertainty of the BC scale ($ZPE_{\rm V}$). This study primarily aims to estimate the amount of the zero-point error of the BC scale if the BC value comes from non-standard bolometric corrections, which are tabulated, or any other source. Consequently, we can assume, just for now, that the first two contributions are zero. Then, equation (7) changes to

$$\frac{\Delta L}{L} = 0.921 \times ZPE_{\rm V}. \tag{9}$$

from which one could obtain the relative uncertainty of L caused by the uncertainty of the zero-point of the BC scale alone; that is, if the absolute visual magnitude and bolometric correction are errorless. Unfortunately, this is not the case nowadays because there are many BC sources giving non-standard BC values. If one of them is used, then it is better not to omit $ZPE_{\rm V}$ in equation (8). Only if one uses a standard BC in equation (5), could $ZPE_{\rm V}$ in equation (8) be omitted, which corresponds to $ZPE_{\rm V}$ being equal to zero; then, there is no need for equation (9). Now the question is how standard and non-standard BC values can be recognized. This is explained in the following section.

5 DEFINITION AND RECOGNITION OF STANDARD STELLAR LUMINOSITIES

5.1 Definition of standard luminosities

We can be certain that the fixed zero-point constant in equation (4) does not cause any uncertainty. Thus, the relative uncertainty $\Delta L/L$ in equation (7) should not include a term implying an uncertainty coming from $C_{\rm Bol}$ because the derivative of a constant is zero by definition.

If and only if the $BC_{\rm V}$ value used in equation (5) comes from a standard source of BC coefficients is it unnecessary to include $ZPE_{\rm V}$ in equation (8). Then, it converts to

$$\Delta M_{\rm Bol} = \sqrt{\Delta M_{\rm V}^2 + \Delta BC_{\rm V}^2}. \tag{10}$$

which means that there are only two error contributions to $\Delta L/L$: the observational random errors of absolute visual magnitude and the error of the BC value itself. Consequently, the L obtained by equation (4), righteously, would be called standard luminosity. The concept of standard BC was originally suggested by Eker et al. (2021).

The term 'standard' or 'non-standard' before the word 'luminosity' would be meaningful if the L of any star is calculated by the method requiring a bolometric correction coefficient. If a standard BC is used, which is already defined by Eker et al. (2021), the computed L is called standard. In contrast, a non-standard BC value makes the computed L non-standard. Standardization of BC values, therefore, is equivalent to the standardization of stellar luminosities. According to the definition of Eker et al. (2021), the tabulated values of $BC_{\rm V}$ could be considered as standard sources for the $BC_{\rm V}$ values if the nominal value of solar absolute bolometric magnitude $M_{{\rm Bol},\odot} = 4.74$ mag and the nominal solar luminosity $L_\odot = 3.828 \times 10^{26}$ W, as used in

$$M_{\rm Bol} = M_{{\rm Bol},\odot} - 2.5\log\frac{L}{L_\odot}, \tag{11}$$

when computing $M_{\rm Bol}$, where L requires observational R and $T_{\rm eff}$ from DDEBs (Eker et al. 2014, 2018). Then, a standard $BC_{\rm V}$ is obtained according to the basic definition of bolometric correction $BC_{\rm V} = M_{\rm Bol} - M_{\rm V}$.

5.2 Recognizing non-standard luminosities

Equations (4) and (11) are both valid for calculating L of a star from its absolute bolometric magnitude. The validity is assured by

$$C_{\rm Bol} = M_{{\rm Bol},\odot} + 2.5\log L_\odot, \tag{12}$$

according to Eker et al. (2021). As equation (4) and the nominal values $M_{{\rm Bol},\odot} = 4.74$ mag and $L_\odot = 3.828 \times 10^{26}$ W were introduced only recently, any other non-standard values of $M_{{\rm Bol},\odot}$ and $L_\odot$ used in computing a BC source (tabulated BC values or $BC_{\rm V}-T_{\rm eff}$ relation) most likely would not produce $C_{\rm Bol} = 71.197\,425\ldots$ mag if L is in SI units, and/or $C_{\rm Bol} = 88.697\,425\ldots$ mag if L is in cgs units, according to equation (12). This is the first and clear indication that there is a zero-point error in the BC value used, which certifies that it is not a standard BC.

It is unlikely, but still a possibility, that the non-standard $M_{{\rm Bol},\odot}$ and $L_\odot$ values would produce $C_{\rm Bol} = 71.197\,425\ldots$ mag if L is in SI units, and/or $C_{\rm Bol} = 88.697\,425\ldots$ mag if L is in cgs units, according to equation (12). This mathematical possibility is unavoidable because there could be an infinite number of $M_{{\rm Bol},\odot}$, $L_\odot$ pairs to produce the same $C_{\rm Bol}$. This is the second type (an unseen indication) of zero-

Table 1. Comparison of BC_V values attributed to the Sun and zero-point constants of bolometric magnitude scale (C_{Bol}) and corresponding $M_{Bol,\odot}$ and $L_\odot$ according to various authors in the near past.

Order (1)	V (mag) (2)	M_V (mag) (3)	M_{Bol} (mag) (4)	BC_V (mag) (5)	$f\,(\times 10^6)$ (erg cm^{-2} s^{-1}) (6)	$F\,(\times 10^{10})$ (erg cm^{-2} s^{-1}) (7)	$L\,(\times 10^{33})$ (erg s^{-1}) (8)	C_{Bol} (mag) (9)	Reference (10)
1	−26.74	4.83	4.75	−0.08	1.36	6.284	3.826	88.70686	Allen (1976)
2	−26.79	4.87	4.74	−0.13	1.37	6.329	3.853	88.70450	Durrant (1981)
3	−26.74	4.83	4.64	−0.19	1.37	6.330	3.850	88.60365	Schmidt-Kaler (1982)
4	−26.76	4.81	4.74	−0.07	1.371	6.334	3.856	88.70534	Bessell et al. (1998)
5	−26.75	4.82	4.74	−0.08	1.367	6.322	3.845	88.70224	Cox (2000)
6	−26.76	4.81	4.75	−0.06	1.368	6.324	3.846	88.71252	Torres (2010)
7	−26.76	4.81	4.75	−0.06	1.361	6.294	3.828	88.70729	Casagrande & VandenBerg (2018)
8	−26.76	4.81	4.74	−0.07	1.361	6.294	3.828	88.69743	Eker et al. (2020)
9	−26.76	4.81	4.645	−0.165	1.361	6.294	3.828	88.60229	This study

point error in the BC values used. Later, in Section 6, we discuss how to treat these two different types of zero-point errors.

For now, let us review some of the contemporary BC_V sources in the near past with different C_{Bol} and corresponding non-standard $M_{Bol,\odot}$ and $L_\odot$ values. Bessell et al. (1998) gave a table for comparing the estimated BC_V of the Sun by various authors, who preferred to adopt $M_{Bol,\odot}$ rather than adopting BC_V of the Sun. Relying on the observed apparent visual magnitude of the Sun (−26.76 mag; Bessell et al. 1998; Torres 2010), (−26.74 mag; Allen 1976; Schmidt-Kaler 1982), (−26.79 mag; Durrant 1981) and adopting either one of the quantities $M_{Bol,\odot}$ or $BC_{V,\odot}$ was inevitable in those years because the zero-point of the BC_V scale was not yet fixed, so it was assumed to be arbitrary. Adopting one of these quantities meant defining a zero-point for tabulated BC_V values. Here we have reconstructed a similar table (Table 1), which enables us to compare various C_{Bol} values as well as the $M_{Bol,\odot}$ and $L_\odot$ values defining it.

The columns of Table 1 are self-explanatory. The sequence number, apparent visual magnitude (a measured quantity), and the absolute visual and adopted absolute bolometric magnitudes of the Sun are given in the first four columns. The BC_V of the Sun as the difference between absolute bolometric and visual magnitudes is in column 5. The corresponding solar fluxes just outside the Earth's atmosphere and on the solar surface are given in columns 6 and 7, while the corresponding solar luminosity is in column 8. Column 9 displays the standard (row number 8) and other figures non-standard zero-point constants (C_{Bol}) according to various BC_V sources. The references are in column 10 in chronological order.

Fig. 1 compares the nominated bolometric zero-point constant and the nominal $M_{Bol,\odot}$ or $BC_{V,\odot}$ values of the Sun to the corresponding values of non-standard sources. A horizontal solid line in Fig. 1(a) marks the nominated zero-point constant (C_{Bol} = 88.697 425... mag) of IAU 2015 General Assembly Resolution B2.

Except for values (3), from Schmidt-Kaler (1982), and (9), which is the test point of this study in Fig. 1(a) with a special $M_{Bol,\odot}$ (4.645 mag) to make all BC values less than zero, the other C_{Bol} values in Fig. 1(a) lie close to the horizontal solid line with various values that are all greater than C_{Bol} = 88.697 425... mag (see Table 1). The largest C_{Bol} (88.712 523) is from Torres (2010). According to Table 1 and Fig. 1(a), the difference between the largest and smallest C_{Bol} is 0.110 mag, which is equivalent to 10.13 per cent uncertainty on the predicted stellar luminosities, even if M_V and BC_V are errorless. The least-deviated C_{Bol} is associated with Cox (2000), which has a C_{Bol} value 0.005 mag bigger than the nominal value, corresponding to a 0.46 per cent error on L.

Fig. 1(b) compares the nominal value of $L_\odot$ (horizontal solid line) to the adopted $L_\odot$ values (data) of the other BC_V sources. The two horizontal dashed lines mark the random observational error limits associated with the nominal solar luminosity, $L_\odot = 3.8275 \pm 0.0014 \times 10^{33}$ erg s^{-1} (see IAU General Assembly Resolution B3). This means that in an ideal case (no error contributions from M_V and BC_V and *ZPE*), a standard stellar luminosity could be as accurate as 0.036 per cent (4 out of 10 000).

Note that the use of a non-standard $L_\odot$ in equations (4) and (11) would produce a systematic error on L even if M_V and BC_V are errorless and no zero-point error exists. The largest of such systematic errors appear as an overestimation of stellar luminosities of about 0.74 per cent if one uses the BC_V values of Bessell et al. (1998), who has the biggest $L_\odot$ in Table 1. The zero-point errors caused only by non-standard $L_\odot$ are apparently less than 0.74 per cent according to Table 1 and Fig. 1(b). The value of $L_\odot$ from Cox (2000) in Table 1 implies a 0.46 per cent zero-point error in Fig. 1(b), which confirms the same amount of error caused by his non-standard C_{Bol} according to Fig. 1(a). According to Fig. 1(c), because Cox (2000) uses $M_{Bol,\odot}$ = 4.74 mag, which is the nominal value, one assumes no zero-point error contribution from it. Despite an error of approximately 0.46 per cent error on L according to Table 1 and Fig. 1, the BC values of Cox (2000) all appear to have been shifted 0.095 mag towards smaller (more negative) values compared to the BC values of Eker et al. (2020).

However, the 0.095 mag systematic shift, creating a difference between the BC_V (max) of Eker et al. (2020) and Cox (2000) means an error of about 8.85 per cent on stellar luminosities. In other words, the BC_V values of Cox (2000) dominate the zero-point errors and propagate within BC_V systematically. Thus, anyone who uses the BC_V values of Cox (2000) will overestimate the stellar luminosities by about 9 per cent, without even including the observational errors and the error of the BC_V value used.

Fig. 1(c) compares the nominal value 4.74 mag (horizontal solid line) to the other adopted $M_{Bol,\odot}$ (data) values of the other BC_V sources. The nominal value is that used by Durrant (1981), Bessell et al. (1998), Cox (2000) and Eker et al. (2020). The non-standard value 4.75 mag is that used by Allen (1976), Torres (2010) and Casagrande & VandenBerg (2018). Another non-standard value, $M_{Bol,\odot}$ = 4.64 mag, is used by Schmidt-Kaler (1982), which is very close to our non-standard trial value of this study (Table 1, Fig. 1c).

It can be concluded here that a non-standard stellar L is easy to recognize. A standard L could be obtained only if standard bolometric correction coefficients were used. One can recognize non-standard bolometric corrections by checking whether nominal $M_{Bol,\odot}$ and

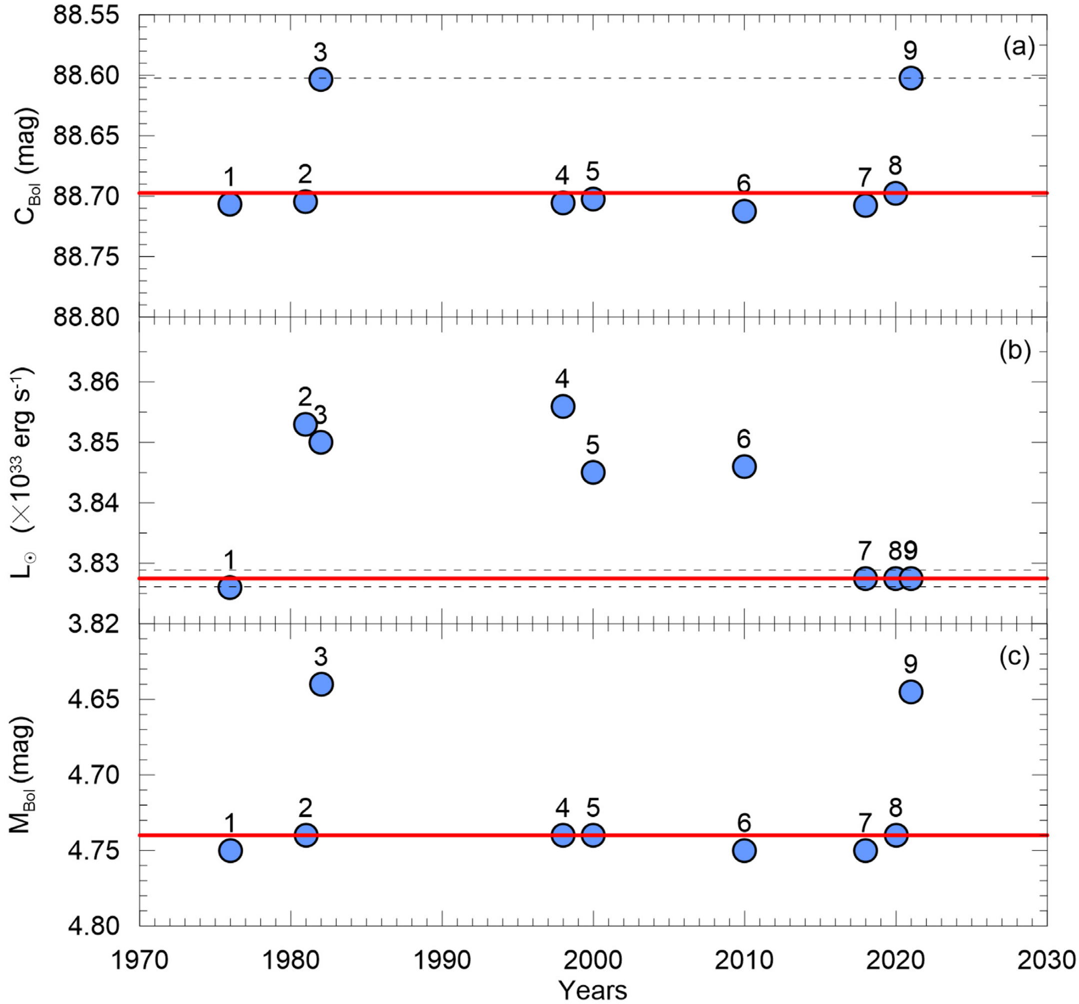

Figure 1. Comparison of nominated bolometric zero-point constant and nominal $M_{\mathrm{Bol},\odot}$ and $L_\odot$ values to the corresponding values of non-standard sources: (1) Allen (1976); (2) Durrant (1981); (3) Schmidt-Kaler (1982); (4) Bessell et al. (1998); (5) Cox (2000); (6) Torres (2010); (7) Casagrande & VandenBerg (2018); (8) Eker et al. (2020); (9) test point used to shift $BC_\mathrm{V}-T_\mathrm{eff}$ curve to make all $BC_\mathrm{V} \leq 0$ mag (this study).

nominal $L_\odot$ were used or not. Although the zero-point of absolute bolometric magnitudes is given as $C_\mathrm{Bol} = M_{\mathrm{Bol},\odot} + 2.5\log L_\odot$, the nominal C_Bol does not necessarily guarantee standardization of the pre-computed BC_V values. On the contrary, a non-standard C_Bol implies non-standard BC_V values. BC_V values could be considered standard if and only if nominal $M_{\mathrm{Bol},\odot}$ and nominal $L_\odot$ are used and if the zero-point of the BC scale (C_2) is calculated as Eker et al. (2021) describes. Authors such as Cox (2000) assume it is arbitrary, thus $C_2 = 0$, in order to make all BC_V less than zero (see Eker et al. 2021, and references therein). This is the case when the zero-point error of the BC scale cannot be estimated from pre-assumed $M_{\mathrm{Bol},\odot}$ and $L_\odot$ but shows itself directly on the BC_V value itself.

6 DISCUSSION

Choosing a non-standard $L_\odot$ from Table 1 contributes very little (less than 1 per cent) to the uncertainty of a computed L. Therefore, the largest error contribution definitely comes from the choice of $M_{\mathrm{Bol},\odot}$. This is because there are infinite numbers of M_Bol and $M_{\mathrm{Bol},\odot}$ pairs to indicate a single value of $L/L_\odot$ according to equation (11). On the process of computing BC_V values, $L/L_\odot$ and $\mathrm{M_V}$ are observatinal quantities coming from DDEBs, so both are fixed values. Therefore, pre-computed BC_V values are affected directly by the choice of $M_{\mathrm{Bol},\odot}$ with the classical method using equation (11). As in the case of Casagrande & VandenBerg (2018), who preferred to use $M_{\mathrm{Bol},\odot} = 4.75$ rather than the nominal 4.74 mag, all pre-computed BC_V values would be 0.01 mag smaller compared with the standard BC_V. Although Casagrande & VandenBerg (2018) do not state clearly which $L_\odot$ value they used, we have assumed they are using the nominal $L_\odot$ as they cite IAU 2015 General Assembly Resolution B3 for the value of $L_\odot$ used in their equation (3) for obtaining the bolometric flux received from a star.

In another aspect, $M_{\mathrm{Bol},\odot}$ acts as the arbitrary zero-point for bolometric magnitudes, as Casagrande & VandenBerg (2018) propose that ‘any value is equally legitimate on the condition that once chosen, all bolometric corrections are scale accordingly’. That is, different authors using different $M_{\mathrm{Bol},\odot}$ end up calculating different BC_V values for the same star. Because $BC_\mathrm{V} = M_\mathrm{Bol} - M_\mathrm{V}$, the zero-point problem shows itself in the produced BC_V values. Using the nominal values of $M_{\mathrm{Bol},\odot}$ and $L_\odot$, Eker et al. (2020) produced a standard $BC_\mathrm{V}-T_\mathrm{eff}$ relation, which has a maximum BC_V value of BC_V (max) = 0.095 mag at $T_\mathrm{eff} = 6897$ K. For this study, we have searched for the value of $M_{\mathrm{Bol},\odot}$, without changing the nominal value of $L_\odot$, which would obtain a $BC_\mathrm{V}-T_\mathrm{eff}$ relation with BC_V (max) =

0.00. We conclude that the answer is $M_{\mathrm{Bol},\odot} = 4.645$ mag; then, all computed BC values will be reduced (more negative) to be less than zero, as listed in Cox (2000). However, such a relation cannot be definitely considered standard.

The problem with the BC_{V} values of Cox (2000) is not the same as adopting $M_{\mathrm{Bol},\odot} = 4.645$ mag in order to make all BC_{V} values negative. His BC_{V} values are all negative, despite his use of $M_{\mathrm{Bol},\odot} =$ 4.74 mag. In fact, his C_{Bol} value is the nearest C_{Bol} among the other sources, which wrongly imply that if the BC_{V} value of Cox (2000) is used, a very accurate luminosity (0.46 per cent) would be obtained, even though the computed L, in reality, has a 9 per cent systematic zero-point error. Most probably, the BC_{V} values of Cox (2000) were calculated using the following definition of BC_{V}:

$$BC_{\mathrm{V}} = 2.5 \log \frac{f_{\mathrm{V}}}{f_{\mathrm{Bol}}} + (C_{\mathrm{Bol}} - C_{\mathrm{V}}) = 2.5 \log \left[\frac{\int_0^{\infty} S_{\lambda}(V) f_{\lambda}\, \mathrm{d}\lambda}{\int_0^{\infty} f_{\lambda}\, \mathrm{d}\lambda} \right] + C_2. \quad (13)$$

Here, C_2, the zero-point constant of the BC scale, was assumed to be arbitrary, so the $C_2 = 0$ value was taken arbitrarily. If C_2 is assumed to be zero, all BC_{V} values become unquestionably less than zero. This is because the visual flux (f_{V}) is never zero but is less than the bolometric flux (f_{Bol}). The logarithm of numbers between zero and one ($f_{\mathrm{V}}/f_{\mathrm{Bol}}$) is always negative, which requires a positive C_2, otherwise (if $C_2 = 0$ or $C_2 < 0$) the BC scale does not have a zero-point; a negative number plus zero or adding two negative numbers does not produce a zero number. The BC producers, who assumed that the zero-point constant of the BC scale is arbitrary, believed that they had the right to make it zero. This way, they unknowingly carried the zero-point error into the BC value itself.

Obviously there are two approaches to remove the zero-point errors. The first approach is that suggested by Casagrande & VandenBerg (2018) and Torres (2010). This entails being cautious when calculating the M_{Bol} of a star from its M_{V} and BC_{V}. Before using the BC_{V} on the formula $M_{\mathrm{Bol}} = M_{\mathrm{V}} + BC_{\mathrm{V}}$, check it out and first learn which $M_{\mathrm{Bol},\odot}$ and $L_{\odot}$ values were used before producing the tabulated BC_{V} values or $BC_{\mathrm{V}}-T_{\mathrm{eff}}$ relation from which the BC_{V} value was taken. Then, it is safe to proceed in calculating $M_{\mathrm{Bol}} =$ $M_{\mathrm{V}} + BC_{\mathrm{V}}$ in the first step. However, do not use equation (4); instead, use equation (11) when calculating L. Do not forget to use the same $M_{\mathrm{Bol},\odot}$ and $L_{\odot}$ values, which you have searched for in order to ensure that they are consistent with the BC_{V} value you are using.

The second approach is suggested in this study. Do not use any value of BC_{V} haphazardly. Use only standard BC_{V} values from standard sources. You can use either one of the equations (4) or (11). It does not matter which, as both are valid for producing the standard L of stars.

Note that the first approach fails to produce a standard L if one uses BC_{V} values from sources such as Cox (2000), which may appear to be using nominal M_{Bol} and $L_{\odot}$ although their BC_{V} values are not standard because they were produced by assuming $C_2 = 0$. Concentrating only on the difference between the M_{Bol} of a star and M_{Bol} of the Sun, the first approach has no answer as to which $L_{\odot}$ was used if the stellar L is taken from published papers where the published L are usually expressed in solar units.

There are no such problems in the second approach.

6.1 Typical accuracy of *L* in method 1 (direct method)

The peak of the histogram distribution of the relative radius errors of DDEBs is 2 per cent, according to Eker et al. (2014). For today's accuracy, we can take it as 1 per cent, because many newly published papers give stellar radii that are even more accurate than ∼1 per cent. According to Eker et al. (2015), typical temperature accuracy is 2–3 per cent. The acceptable stellar effective temperature uncertainty for single stars in general is 1–2.5 per cent, according to Masana, Jordi & Ribas (2006). Because the direct method uses the effective temperatures of DDEBs, we may find a typical temperature uncertainty of 2–3 per cent. Consequently, equation (1) indicates that the typical uncertainty range of stellar L is 8.2–12.2 per cent. On the more accurate side, there could be stars such as the primary component of V505 Per that have $\Delta R/R = 1.09$ per cent and $\Delta T_{\mathrm{eff}}/T_{\mathrm{eff}} = 0.32$ per cent with corresponding $\Delta L/L = 2.53$ per cent (Tomasella et al. 2008). That is, the accuracy of a few percentage levels has already been achieved by the direct method.

6.2 Typical accuracy of *L* in method 2 (using MLR)

Among the three methods, the method using the MLR is the least accurate. This is because the relative uncertainty of $L(\Delta L/L)$ is determined by the standard deviation (SD) of stellar luminosities on a $\log M - \log L$ diagram according to equation (3), where SD could be different at different mass ranges (Eker et al. 2015, 2018). The most accurate mass range, which was called the ultra-low-mass domain covering stellar masses in the range $0.179 < M/\mathrm{M}_{\odot} < 0.45$, has $SD = 0.076$ mag which corresponds to $\Delta L/L = 17.5$ per cent. The most dispersed range, which is called the high-mass domain covering stellar masses in the range $2.4 < M/\mathrm{M}_{\odot} \leq 7$, has $SD = 0.165$ mag, which corresponds to $\Delta L/L = 37.99$ per cent (Eker et al. 2018).

6.3 Typical accuracy of *L* in method 3

6.3.1 *Typical accuracy of L using a standard* BC

Standard stellar luminosities are those calculated by the method using a pre-determined standard BC_{V}. The most important problem with this method is that the systematic zero-point errors of non-standard BC_{V} could be as big as 0.11 mag, corresponding to 10.13 per cent errors in the predicted stellar luminosities. Such systematic errors could be removed if and only if standard BC_{V} sources are used. To make the comparison meaningful, here we assume that the zero-point error has already been removed or taken to be zero.

A typical accuracy of standard L, therefore, could be calculated by equations (7) and (8) but taking ZPE_{V} as zero in equation (8).

There could be three contributions to the uncertainty of ΔM_V according to equation (6). Eker et al. (2020) gives

$$\Delta M_V = \sqrt{(\Delta m_V)^2 + \left(5 \log e \frac{\sigma_{\varpi}}{\varpi}\right)^2 + (\Delta A_V)^2}, \quad (14)$$

where the first term in the square root stands for the error contribution from the apparent brightness, the second term represents the contribution from stellar parallaxes, and the last term is for the contribution from interstellar extinctions. Nowadays, apparent magnitude uncertainties are in the order of millimagnitudes. If we assume extinction errors (ΔA_{V}) are about 0.01 mag, the parallax errors would definitely dominate the others; if there is a 10 per cent error on the parallax, $\Delta M_{\mathrm{V}} = 0.217$ mag. For stars with a 5 per cent error in their parallaxes, $\Delta M_{\mathrm{V}} = 0.109$ mag. The histogram distribution of the parallaxes for 206 DDEBs (400 stars), from which the standard BC_{V} was extracted

(Eker et al. 2020), has a peak (median) of 2 per cent. If we take this as a typical parallax error, then typically $\Delta M_{\rm V} = 0.044$ mag.

The typical standard error of a $BC_{\rm V}$ value, if it comes from a standard $BC_{\rm V}-T_{\rm eff}$ curve, would be of the order of 0.011 mag (Table 2) for the range of temperatures (3000–36 000 K) considered for main-sequence stars by Eker et al. (2020). The re-arranged data indicate that a standard error of $BC_{\rm V}-T_{\rm eff}$ is reduced to 0.009 mag for medium temperatures (5000–10 000 K) while it is 0.59 mag for lower temperatures, and 0.028 mag for higher temperatures. Inserting typical $\Delta M_{\rm V} = 0.044$ mag and a typical standard error of a BC (0.009 mag) into equation (10), a typical $\Delta M_{\rm Bol} = 0.045$ mag is obtained for the middle temperatures. Inserting this into equation (7), a typical $\Delta L/L$ becomes 4.14 per cent.

Now, let us assume the extreme case of a star (CM Dra) with a relative parallax error of 0.050 per cent (Brown et al. 2021). We can assume no extinction because it is only 14.86 pc away, according to *Gaia* eDR3 data. Then, the millimagnitude accuracy of its apparent brightness would imply $\Delta M_{\rm V} = 0.0019$ mag. Assuming its $BC_{\rm V}$ has a standard error of 0.011 mag (last row in Table 2), its standard luminosity would have an uncertainty of 1.03 per cent. Accuracy in standard luminosity even increases to 0.82 per cent for the middle temperatures.

Here, we run into an unexpected case: the secondary method, using a BC, provides more accurate stellar luminosities than the direct method. It is too good to be true. Apparently, the problem must be taking the standard error of a standard $BC_{\rm V}-T_{\rm eff}$ curve as the standard error of a BC value before computing $\Delta M_{\rm Bol}$.

As Eker et al. (2021) stated, a standard $BC_{\rm V}-T_{\rm eff}$ curve obtained from $BC_{\rm V}$ and $T_{\rm eff}$ values is similar to MLRs obtained from masses and luminosities, but not like the Planck curve representing the spectral energy distribution (SED) of a star. Thus, the standard error of the MLR (curve) cannot be used as the standard error of an L for a given M. This is because the scattering of data from the relation is not only due to observational errors but also due to the different ages and chemical compositions of the stars on the $M-L$ diagram. Therefore, $\Delta L/L$ should be calculated directly from the standard deviations, as described in Section 3.2.

Similarly, one must not use standard errors (column 3 of Table 2) but use the standard deviations (column 2 of Table 2) when computing typical errors of $M_{\rm Bol}$. If this is done, 0.215 mag standard deviation for the total range indicates $\Delta L/L = 0.198$, and 0.142 mag standard deviation for the middle temperatures indicates $\Delta L/L = 0.131$ for the star CM Dra, which has $\Delta M_{\rm V} = 0.0019$ mag. For a typical $\Delta M_{\rm V} = 0.044$ mag, $\Delta L/L$ becomes 20.2 per cent for all main-sequence temperatures in general, and 13.7 per cent for the middle temperature only. Therefore, the indirect method of using standard BC when computing standard stellar luminosities could be considered better than using the MLR, but not as good as the direct method.

6.3.2 *Typical accuracy of L using a unique* BC

Another advantage of the method of computing the standard L of a star using its BC is when one does not need a pre-computed $BC_{\rm V}-T_{\rm eff}$ curve or a table giving standard BC values. The unique BC of a star can be obtained directly from its SED according to equation (13). Using this equation, however, requires spectroscopic observations of stars covering a sufficient spectral range, at least more than the photometric filter, which is used to determine its absolute magnitude in equation (6). Interstellar reddening is still there to spoil the SED, but it is always possible to restore the SED, as done by Stassun & Torres (2016). This might not even be necessary if the star is in the Local Bubble where interstellar extinction can be ignored.

Table 2. Standard deviation of data from the $BC_{\rm V}-T_{\rm eff}$ curve of Eker et al. (2020) and standard errors of $BC_{\rm V}$ values.

$T_{\rm eff}$ range (K)	N (1)	SD (2)	$SD/\sqrt{N}$ (3)
3000–5000	35	0.349	0.059
5000–10 000	261	0.142	0.009
10 000–36 000	104	0.290	0.028
3000–36 000	400	0.215	0.011

Assuming that C_2 is a well-defined quantity (Eker et al. 2021), the error of BC in this method depends on how accurately $f_{\rm V}/f_{\rm Bol}$ can be determined. If the quantity $f_{\rm V}/f_{\rm Bol}$ is determined at a level of a few per cent, as there is no error contribution from C_2, then according to equation (5) the accuracy of $M_{\rm Bol}$ depends on the accuracy of two quantities, $M_{\rm V}$ and $BC_{\rm V}$, which means that the $M_{\rm Bol}$ of a star could be obtained at a low percentage. For example, using typical $\Delta M_{\rm V} = 0.044$ mag as pointed out in Section 5.3.1, assuming $\Delta BC_{\rm V} = 0.03$ mag, $\Delta M_{\rm Bol}$ is 0.053 mag which means $\Delta L/L$ is 4.9 per cent. If a star has a very accurate parallax, as CM Dra does, then its absolute visual magnitude could be as accurate as 0.0019 mag, and if its BC is calculated with an accuracy of 1 per cent, then its luminosity would be 0.9 per cent.

We estimate here that the typical accuracy of a standard stellar luminosity could be of the order of a few per cent if the method uses the star's unique BC, which can be computed from its SED. For ideal cases, it could be as accurate as 1 per cent or even more, depending upon the accuracies of M_V and the unique BC itself. Therefore, using a unique BC that is computed from its SED is a real advantage over all other methods, including the direct method.

7 CONCLUSIONS

The accuracy of predicted stellar luminosities using the direct method and two secondary methods has been revised and a new concept, 'standard stellar luminosity', has been defined. The luminosities that were calculated by the direct method from observational radii and effective temperatures are more accurate than the luminosities estimated by the secondary methods, which require the MLR or the pre-computed standard BC. The luminosities produced by the direct method have been shown to be accurate from about 8.2 per cent to 12.2 per cent, and they could be more accurate by 2–3 per cent. Stellar luminosities obtained by the direct method are standard by definition because there can be only one luminosity for a star of given radius and effective temperature.

The stellar luminosities by the method using the MLR are the least accurate. Depending upon the mass range where the classical MLR operates in the form $L \propto M^{\alpha}$, they could be most accurate at about 17.5 per cent (for low-mass stars; $0.179 \leq M/{\rm M}_{\odot} \leq 0.45$) and least accurate at about 38 per cent (for high-mass stars; $2.4 \leq M/{\rm M}_{\odot} \leq 7$). Luminosities calculated using a method with the MLR cannot be called standard because, first of all, a standard MLR has not been defined yet, and perhaps it never will be. This is because an MLR in the form $L \propto M^{\alpha}$ is the relation between the typical mass and typical luminosity of a main-sequence star belonging to a certain region (e.g. Galactic disc stars in the solar neighbourhood). Moreover, there is no relation or process providing the true luminosity of a main-sequence star from its mass except a standard stellar structure and evolutionary model (not yet defined or agreed upon), which naturally would include the chemical composition and age, as with the other independent parameters.

The stellar luminosities produced by the method using a pre-determined *BC* are more accurate than the luminosities computed by the method of using the MLR, but less accurate than the luminosities produced by the direct method. It has been shown that the method that uses a standard *BC* would provide 13.7 per cent accuracy in predicted luminosities if the typical accuracy of the absolute visual magnitude is about ±0.044 mag for main-sequence stars with middle temperatures of $5000 \leq T_{\rm eff} \leq 10\,000$ K. Depending upon other factors, such as the accuracy of its parallax, interstellar extinction and accuracy of the standard *BC*, the accuracy is either improved or becomes worse. It has been said that such luminosities are called standard only if the *BC* value used in computing it is a standard *BC*. Non-standard *BCs* produce non-standard luminosities.

Note that if a *BC* value is from a tabulated standard *BC* table and/or from a standard $BC_{\rm V}-T_{\rm eff}$ relation, the *BC* value cannot be called unique. Tabulated standard *BC* tables and/or $BC_{\rm V}-T_{\rm eff}$ relations in the literature do not provide unique *BC* values because they have already been produced from pre-computed unique *BC* values; thus, they provide only a mean value to represent stars of a given effective temperature with different ages and chemical compositions.

Therefore, a unique *BC* is a *BC* computed according to equation (13) for a star from its spectrum with sufficient spectral coverage and resolution. Unlike tabulated tables and/or $BC_{\rm V}-T_{\rm eff}$ relations where chemical composition and age information has been lost, a computed $f_{\rm V}/f_{\rm Bol}$ from the well-observed spectrum of a star retains this information; therefore, such computed *BC* values of stars are unique. Using unique *BC* when computing standard luminosity has the advantage of providing an even more accurate standard *L* than the other methods, including the direct method.

Unfortunately, this new method, described in the present study, has no application yet because it requires C_2 (equation 13) for the *V* band and other bands, which are not available in the literature (Eker et al. 2021). Therefore, we encourage the determination of C_2 values for various bands first, and then the determination of $f_{\rm V}/f_{\rm Bol}$ from observed stellar spectra in order to compute a unique $BC_{\rm V}$ value for each star. Then, unique stellar luminosities could be computed using equations (4), (5) and (6).

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DATA AVAILABILITY

The data underlying this article are available at https://dx.doi.org/10.1093/mnras/staa1659

REFERENCES

Allen C. W., 1976, Astrophysical Quantities, 3rd edn. Athlone Press, London
Andersen J., 1991, A&AR, 3, 91
Bakış V., Bakış H., Eker Z., Demircan O., 2007, MNRAS, 382, 609
Benedict G. F. et al., 2016, AJ, 152, 141
Bessell M. S., Castelli F., Plez B., 1998, A&A, 333, 231
Bressan A., Marigo P., Girardi L., Salasnich B., Dal Cero C., Rubele S., Nanni A., 2012, MNRAS, 427, 127
Brown A. G. A. et al. (Gaia Collaboration), 2021, A&A, 649, A1
Casagrande L., VandenBerg D. A., 2018, MNRAS, 475, 5023
Cayrel R., Castelli F., Katz D., van't Veer C., Gomez A., Perrin M.-N., 1997, ESA SP-402, Proc. ESA Symp. *Hipparcos* - Venice 97. ESA Publications Division, Noordwijk, p. 433
Cester B., Ferluga S., Boehm C., 1983, Ap&SS, 96, 125
Chen Y. et al., 2019, A&A, 632, A105
Code A. D., Bless R. C., Davis J., Brown R. H., 1976, ApJ, 203, 417
Cox A. N., ed. 2000, Allen's Astrophysical Quantities. AIP Press, New York
Demircan O., Kahraman G., 1991, Ap&SS, 181, 313
Durrant C. J., 1981, in Hellwege K. H., ed., Landolt–Börnstein: Numerical data and functional relationships in science and technology, New Series, Vol. VI/2A. Springer, Berlin, p. 82
Eddington A. S., 1926, The Internal Constitution of the Stars. Cambridge University Press, Cambridge
Eggen O. J., 1956, AJ, 61, 361
Eker Z., Bilir S., Soydugan F., Yaz Gökçe E., Soydugan E., Tüysüz M., Şenyüz T., Demircan O., 2014, PASA, 31, e024
Eker Z. et al., 2015, AJ, 149, 131
Eker Z. et al., 2018, MNRAS, 479, 5491
Eker Z. et al., 2020, MNRAS, 496, 3887
Eker Z., Bakış V., Soydugan F., Bilir S., 2021, MNRAS, 503, 4231
Fernandes J., Gafeira R., Andersen J., 2021, A&A, 647, A90
Flower P. J., 1996, ApJ, 469, 355
Gafeira R., Patacas C., Fernandes J., 2012, Ap&SS, 341, 405
Girardi L. et al., 2008, PASP, 120, 583
Gorda S. Y., Svechnikov M. A., 1998, Astron. Rep., 42, 793
Green G. M., Schlafly E., Zucker C., Speagle J. S., Finkbeiner D., 2019, ApJ, 887, 93
Griffiths S. C., Hicks R. B., Milone E. F., 1988, J. Royal Astron. Soc. Canada, 82, 1
Hadrava P., 1995, A&AS, 114, 393
Heintze J. R. W., 1973, in Hauck B., Westerlund B. E., eds, Proc. IAU Symp. 54, Problems of Calibration of Absolute Magnitudes and Temperature of Stars. Reidel, Dordrecht, p. 231
Henry T. J., 2004, in Hilditch R. W., Hensberge H., Pavlovski K., eds, ASP Conf. Ser. Vol. 318, Spectroscopically and Spatially Resolving the Components of the Close Binary Stars. Astron. Soc. Pac., San Francisco, p. 159
Henry T. J., McCarthy D. W., 1993, AJ, 106, 773
Hertzsprung E., 1923, Bulletin of the Astronomical Institutes of the Netherlands, 2, 15
Ibanoğlu C., Soydugan F., Soydugan E., Dervişoğlu A., 2006, MNRAS, 373, 435
Johnson H. L., 1964, Boletin de los Observatorios de Tonantzintla y Tacubaya, 3, 305
Johnson H. L., 1966, ARA&A, 4, 193
Kuiper G. P., 1938a, ApJ, 88, 472
Kuiper G. P., 1938b, ApJ, 88, 429
Lallement R., Babusiaux C., Vergely J. L., Katz D., Arenou F., Valette B., Hottier C., Capitanio L., 2019, A&A, 625A, 135
Leroy J. L., 1993, A&A, 274, 203
Malagnini M. L., Morossi C., Rossi L., Kurucz R. L., 1985, A&A, 152, 117
Malkov O. Y., 2007, MNRAS, 382, 1073
Masana E., Jordi C., Ribas I., 2006, A&A, 450, 735
McCluskey G. E., Jr, Kondo Y., 1972, Ap&SS, 17, 134
McDonald J. K., Underhill A. B., 1952, ApJ, 115, 577
McLaughlin D. B., 1927, AJ, 38, 21
Moya A., Zuccarino F., Chaplin W. J., Davies G. R., 2018, ApJS, 237, 21
Petrie R. M., 1950a, Publications of the Dominion Astrophysical Observatory, 8, 341
Petrie R. M., 1950b, AJ, 55, 180
Popper D. M., 1959, ApJ, 129, 647
Russell H. N., Adams W. S., Joy A. H., 1923, PASP, 35, 189
Schlafly E. F., Finkbeiner D. P., 2011, ApJ, 737, 103

Schmidt-Kaler Th., 1982, in Schaifers K., Voigt H. H., eds, Landolt-Bornstein: Numerical Data and Functional Relationships in Science and Technology, Vol. VI/2B. Springer, Berlin
Stassun K. G., Torres G., 2016, AJ, 152, 180
Strand K. A., Hall R. G., 1954, ApJ, 120, 322
Sung H., Lim B., Bessell M. S., Kim J. S., Hur H., Chun M., Park B., 2013, J. Korean Astron. Soc., 46, 103
Tomasella L., Munari U., Siviero A., Cassisi S., Dallaporta S., Zwitter T., Sordo R., 2008, A&A, 480, 465
Torres G., 2010, AJ, 140, 1158
Torres G., Andersen J., Giménez A., 2010, A&AR, 18, 67

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