EVOLUTION OF THOUGHT FOR PRODUCTION FUNCTION: FROM CLASSICAL ECONOMICS TO NON-PARAMETRIC FUNCTIONS

Ast. Prof. Can BEKAROĞLU[1]

ABSTRACT

This paper tackles the evolution of thought regarding the production theory from classical to neoclassical era, reaching to its modern techniques in DEA (Data Envelopment Analysis). David Ricardo, with his seminal book, "On the Principles of Political Economy", is a great example depicting the classical chain of thought of his time, while the famous Cobb-Douglas Function highlights the shift in economical thinking from "average" to "marginal" and from "macro to "micro" and from "labor focused" to "capital & labor". Based on the footsteps of the neoclassical assumptions, non-parametric approaches such as DEA evolve on the concept of producing a certain output level from the given input bundle, and derive their frontier from the given data with the defined technology. DEA literature on undesirable outputs extends by introducing solutions to well known issues, such as the relaxation of the "free-disposability" for certain outputs, along with the differentiation between weak and strong disposability, asymmetric treatment of bad and good outputs, and the use of shadow prices.

Keywords: Classic Economics, Neoclassical Economics, Cobb-Douglas, DEA

JEL Classification: B10, B21, C00

KLASİK İKTİSATTAN NON-PARAMETRIK FONKSİYONLARA, ÜRETİM FONKSİYONUNA DAİR İKTİSADİ DÜŞÜNCENİN EVRİMİ

ÖZET

Bu makale, klasik iktisadi çagdan neo-klasik çağa, oradan da DEA (Veri Zarflama Analizi) gibi günümüzün modern non-parametrik tekniklerine uzanacak şekilde üretim fonksiyonunun ekonmik düşünce bazındaki evrimini ele almaktadır. David Ricardo, "On the Principles of Political Economy" adlı özgün çalışmasıyla, klasik iktisadi düşünce yapısını özetlerken, ünlü Cobb-Douglas Fonksiyonu, iktisadi anlayıştaki "ortalama"dan "marjinal"e, "makro"dan "mikro"ya, "emek"ten "emek ve sermaye"ye geçişi betimlemektedir. Neoklasik temeller üzerine bina edilen, DEA gibi non-parametrik yaklaşımlar, mevcut girdi sepeti ile belli bir çıktı seviyesi elde etmek ve eldeki veriler ile mevcut teknolojiye göre üretim sınırlarını türetmek üzere geliştirilmişlerdir. İstenmeyen çıktıları da içine alacak şekilde gelişen DEA literaturü, belli çıktılar için serbest atılabilirlik (free-disposability) varsayımının kaldırılabilmesi, hatta zayıf ve güçlü atılabilirlik (weak-strong disposability) kavramları ayrımı, kötü ve iyi çıktıların asimetrik ele alınması ve gölge fiyatların kullanımı ile bilinen birçok soruna çözüm bulmuştur.

[1] Ast. Prof. Can Bekaroğlu, ÇanakkaleOnsekizMartUniversity, Faculty of Political Sciences, Department of Economics, orcid: 0000-0002-4033-1897

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1. INTRODUCTION AND THE RICARDIAN PRODUCTION FUNCTION

The production function can simply be described as the framework that defines the relationship between inputs and outputs in the process of production, which is basically the transformation of those inputs into outputs given the available technology. However, the intricacies and the structure of this relationship, as well as the desire to find a more formalized, encompassing and unified technical definition for it has been in the works at least since the times of David Ricardo, who, in his seminal book titled "On The Principles of Political Economy" (Ricardo, 1821, Chapter 1 pages 1-48) required first to understand the concept of "value", giving two definitions for it:

a) The Use Value, which depends on the utility derived from the use of a good, and will determine the demand for that good. In this sense, water is very valuable since the average benefit derived from it is very high. The Use-Value, in Ricardo's understanding, is constant. However, even basic goods like water, have changing benefits, from time to time, person to person, and at different quantities. In Ricardian Language, there is a homogeneous utility derived from a good.

b) The Exchange Value, which depends on the quantity of labor necessary to produce a good, is also the relative value in the market in exchange with other goods. Namely, the cost of a good (or rather its supply) dictates the exchange value of a good in the long run, although the short-run values might fluctuate depending on demand.

One major flaw for such explanation is illustrated by Ricardo himself. Supply constrained goods like diamond have many times more exchange value than most other goods that require higher amount of labor to be processed, so the cost (in labor terms) cannot be the reason for its high exchange value. Therefore supply is not solely determined by cost (or labor-cost for that matter).

Ricardo takes "labor" as the absolute yard-stick in determining the value of goods, since its (exchange) value, in his perspective, does not change in the long run. All other goods, and even capital, change in its value, depending on the quantity of labor used to produce them, while labor itself is not produced but taken as a direct input from

individuals. This implicitly assumes that the product of labor is homogeneous once it is standardized with given intensity and skills-set.

Although Ricardian value of production is measured in labor terms, the wages paid to the workers are not necessarily equal to the value of their work, but rather determined by the labor supply, which is ever abundant. Wages, thus, tend to be equal to the subsistence level in the long run, with the effect of the adjusting supply due to the population changes and short-run fluctuations in workers' welfare. The discrepancy between the wages and labor's value of production enables the capital owner to induce profits, though after due “rents” are paid to the land-owners.

The rents, in Ricardian perspective, arise from “homogeneous” profits across non-homogeneous land, exhibiting variable returns to scale. Without the existence of such variation, the rents would be effectively zero, and only in the existence of such unequal distribution of land, rents would be granted to the relatively more productive land owners. Therefore rising rents must be a symptom of rising inequality.

Ricardo explains that the production must equal the sum of those three factors: rent paid to the land-owners and determined by the unequal distribution of land; wages paid to the workers, determined by the subsistence level; and profit as the leftover share for the capital. Positive profit will lead to capital accumulation and better use of capital, possibly increasing its productivity, and requirement of decreasing quantities of land (under constant demand), which will lead to a change in the distribution of income.

In Ricardian framework, what makes land eligible for rent is its “scarcity” and “non-uniformity”. Without either of those constraints, rent would be effectively zero. It is, rather, the non-uniformity aspect of land, causing diminishing returns to productivity, which is the crucial point here. The reason for such a result is the non-homogeneity of the land, which still implicitly assumes the same land will always yield the same product regardless of how many times it is plowed under ceteris paribus.

Ricardo does not talk about “marginal product” of land, even between heterogeneous pieces, but the differences between such different pieces of land might be interpreted as the marginal product of a relatively more fertile land upon another less fertile one. On the other hand, labor was not considered to exhibit diminishing returns to

production since it was not scarce and could have easily been replaced by another, thus forced to behave identically.

2. COBB-DOUGLAS AND THE NEOCLASICAL PRODUCTION FUNCTION

The seminal paper titled "*A Theory of Production*" by Cobb & Douglas in 1928 shows the establishing trends of the Neoclassical Production Theory at the time, depicting both labor and capital as the key components of the production function, which may not seem so different from the classical school at first sight. However, the roles that are assigned to the capital and labor, and the way they interact with the production process are vastly different.

Cobb & Douglass assume that production is the result of three key components, namely capital, labor and technology, which is determined exogenously. Total Factor Productivity (TFP), in this framework, covers not only technological improvements but all other changes in the production function that lead to an increase (or decrease) in the production not captured by the changes in labor and capital. In some ways, it is the slack variable determined exogenously.

Although such assumptions might be logical for small economies that adopt only imported technologies without much friction or adaptation, it fails short of expectations of an actual endogenous production where the technology is determined within the system. Nevertheless, the Cobb & Douglas paper did a phenomenal job explaining several principles:

Marginal Productivity of Capital and Labor: The paper clearly shows that both labor and capital exhibit diminishing marginal returns to productivity separately but constant returns to a scale as a whole. Therefore there is a natural limit to how much capital and labor will be used in the production, depending on their marginal productivities.

Factor Prices: Both wages and interest rates equal the marginal productivity of their factors in factor markets. Increased marginal labor and capital productivities will simply increase their factor demands and help determine market prices with the given supplies.

One major difference between the classical and neoclassical views stems from the periods that shaped people's minds. An agriculture dominant society was focused on land, with little capital accumulation, while an industrialized society is much more capital abundant, with a shifting focus from land to capital and entrepreneurship. Thus the scarcity of land is not as pronounced, and the increased capital has striking consequences both on labor productivity, and the capital's role as a major instrument in production.

3. COMPARING THE RICARDIANAND NEOCLASSICAL VIEWS

Ricardo took labor as the yard-stick of production, with its seemingly solid and stable value explaining all other goods, including the means of production. Land is subject to rent due to its non-homogeneous nature, and is used for production only after labor input. Capital, likewise, is used in production thanks to labor input. Labor might change in productivity but that is only because of the variation of capital's own productivity. In other words, the productivity change happens at average level, and only from capital to labor, not the other way around since labor's inherent value and productivity are assumed constant.

Cobb-Douglas, on the other hand, show that the interaction between labor and capital is a two-way relationship, since both labor and capital do influence each other. In this sense, labor's value is not constant either but subject to change, depending on the capital accumulation. Cobb-Douglas also showed that this interaction does not happen on the “average” but rather on the “marginal” level. It is because marginal productivity of labor has increased so much that (the capital per labor rose to 2.5 times of its original level between 1899 and 1922) the wage rate is no longer equal to the subsistence level. The decrease in relative abundance in labor also helps explaining the argument, which is a result of labor demand increasing much faster than the supply.

Ricardo, although talks about diminishing returns with respect to non-homogeneous land, fails to recognize the existence of diminishing returns with respect to the over-use of homogeneous capital and labor, which would require the use of marginals, rather than averages. Though he knew that the changes in capital productivity affect the labor productivity, he considered more of a constant and homogeneous increase in all labor, regardless of the amount of use. In neoclassical

view, however, this acts like a shift in all marginal levels, which is also a two way relationship between labor and capital. Furthermore, those cross interactions are subject to diminishing returns. In other words, an increase in capital has a positive but diminishing role in the marginal productivity of labor and vice versa.

The Cobb-Douglas Function, although phenomenally helpful in explaining the factors of production and their marginal contributions, is fatally simplistic and flawed in its mechanism, taking the technological progress as "manna from heaven", which is further explained by endogenous production and growth models more recently.

Another significant difference between classical and neoclassical schools is the utilization of mathematics in their language. Ricardo, although uses math to express his ideas and explain the logic as to why and how things happen in economics, his language is simple enough to be understood for an "average" person who is good at logical thinking but not necessarily savvy at math. Neoclassical thinking directly addresses to the people who literally think at the "margin", with a good level of familiarity with mathematics, where intuition takes a step back behind the curtain.

Although a well written mathematical equation can express the ideas of many pages in an extremely efficient manner, rather than overly profuse and philosophical explanations of the classical era, it might also estrange masses who have a knee jerk reaction towards serious math. It is the neoclassical mathematical approach though, which enabled the marriage of statistical sciences to beget parametric and nonparametric methods such as Econometrics and Data Envelopment Analysis, which inevitably contribute to the economics theories to further improve, especially in quantitative analysis.

4. PRODUCTION FUNCTION AND THE TECHNICAL INEFFICIENCY

One interesting aspect of the Cobb-Douglas study is to synthesize a hypothetical production function from the given input data, with estimated contributions for each, and decide the exogenous total factor productivity thereof. Although highly useful, this approach implicitly assumes that the changes in the total factor productivity are inherently exogenous and does not properly address whether or not such changes are due to a shift in the production function or simply a movement closer towards a virtually optimum frontier.

Debreu's 1951 Paper, “The Coefficient of Resource Utilization”, reveals the fact the Cobb Douglas Production implicitly assumes that all resources are used efficiently and the economy operates at the frontier, while total factor productivity (TFP) behaves like a productivity shock that shifts the production function. Debreu suggests that the economy is not necessarily operating on the pareto-optimum frontier but usually inside at suboptimal levels; and with the given technology, the changes in the so called TFP arise mainly due to three major causes that result in sub-optimal production, namely

a) Underutilization of Resources,

b) Technical Inefficiency,

c) Inefficiency of Economic Organization,

All of these lead to what Debreu call “Dead Loss”. Therefore, without even a technological breakthrough or changes in factors of production, the output can increase possibly due to changes in the factors above, which is technically a movement closer to the frontier where the pareto-optimum levels lie.

Farrell, in his 1957 Paper, “The Measurement of Production Efficiency” argues that the stylized production functions such as Cobb-Douglas are usually wildly optimistic, not suitable to a real life scenario, while obtaining real life data from non-parametric analysis is both more realistic and viable, and does not contradict any of the neoclassical principles.

He notes that previous measures of efficiency usually take one input as the yardstick (usually the labor employed) ignoring all other inputs, leading to inconsequential results. He then chooses to go on the footsteps of Debreu, assuming that the actual production is usually a fraction of the maximum attainable amount (the production frontier), where the firms have an efficiency level equal to or less than 1.

Farrell talks about two types of efficiency; “Technical Efficiency”, which is similar to Debreu's “Coefficient of Resource Allocation”, where firms produce a certain fraction of the maximum attainable output given the obtained data, taking a value between 0 and 1. Technical efficiency might be obtained with many different input-bundles and this is where the “Price Efficiency” comes into play. Firms are price efficient when they choose the cost-minimizing input-bundle. So technical efficiency

does not automatically imply price efficiency, and technically efficient firms might still be price inefficient.

The main problem in this approach is to estimate an isoquant production curve from observed data points. The isoquant here, by definition, is “convex to the origin”, and “has nowhere a positive slope”, making this isoquant (SS) the most conservative estimate. In other words, it depicts the minimum output level that needs to be produced with the 2 input-bundle to be considered efficient. Addition of new observations may decrease the efficiencies but will never increase them, which is expected since a good college by local standards, for instance, may be only average by national standards but not necessarily great, due to a greater pool of data. It also allows for diseconomies of scale and can be influenced by the measurement methods.

All of the lines on such an isoquant, with the given output are technically efficient, although only the cost-minimizing point on this isoquant is price-efficient. However, price efficiency here implicitly assumes perfectly elastic supply and substitutability of each factor similar to the Cobb-Douglas form (with respect to capital & labor) leading to under-efficiency in terms of price.

Homogeneity is another problem constructing the efficient productive frontier. If all firms suffer a systematic inefficiency, or are composed of homogeneous firms, then their technical efficiencies will be overstated. Certainly a bigger set of observations always help, but the efficiency of a relatively more diverse group will appear lower but more realistic, while the pooled data will offer even better results since the technique is highly dependent on the data set.

5. COMPARING THE COBB-DOUGLAS WITH FARRELL & DEBREU PAPERS

Cobb-Douglas correctly identified capital, labor, and technology as the three components of production on a consistent neoclassical framework, each contributing with their marginal productivities equal to their factor prices. It was a progress by leaps and bounds on the classical theory, but still far from perfect due to its well known setbacks.

First of all, Cobb-Douglas uses a specific production function ($AK^{\alpha}L^{\beta}$), where A stands for exogenous total factor productivity, while the marginal productivities of

capital and labor are given by α and β. Although it is reasonable to assume that α and β are subject to change with the observed data, the functional form is just assumed and overly-simplistic with all the non-fitting slack swept under "A". We have no guarantee that this is the right functional form as it is plausible to come up with an entirely different function that yields similar or even better fit with the data.

Farrell's technique, on the other hand, does not need or use any specific functional form, but instead constructs a frontier based on the observations, effectively nullifying such a possibility. The elimination of a specific form does not defy the neoclassical principles though, since variable returns to scale are allowed, along with convexity of the input bundle and free disposability (which was later coined). Even though we do not have functional form, we have a clear relationship between the input and output bundles, creating a virtual production frontier.

Secondly, Cobb-Douglass implicitly assumes all agents operate at the production frontier, producing optimal outputs. The changes in TFP (A) are directly reflected as a shift in the production frontier, although they may very well be an improvement on the existing technology. Debreu (1951) argues that the agents do not necessarily operate at the frontier but inside at suboptimal levels, and apart from the real technological shocks that will genuinely shift the production frontier, most of the changes in TFP come from increases in resource utilization, as well as technical and organizational efficiency levels.

Third, the exogeneity of TFP is a very well known deficiency of the Cobb-Douglass function. Technology is not a separate input used in the production or exogenous effect that dictates the efficacy of your production, but rather an integral part of the production engine that determines the very functional form and how it interacts with the used inputs. So a change in technology yields a change in the whole functional form and its interaction with inputs. Although more sophisticated endogenous models have been developed by time, they are still prone to similar limitations due to a specific functional form, while Farrell's nonparametric methodology circumvents this issue by avoiding a specific function but rather relying entirely on the observed data, where technology is inherently integrated, and systematic efficiency differences are likely to be due to technological or organizational discrepancies.

Finally, Farrell's technique is able to draw an efficiency figure based on multiple inputs, rather than one input such as labor ignoring all others. Using labor productivity as the sole basis for efficiency possibly made sense in a very agricultural society, but after industrialization along with massive capital accumulation, it makes little sense with such discrepancy of capital spread. Cost comparisons are also of limited use due to price diversity across regions. Above all, Farrell is able to distinguish between price efficiency and technical efficiency in spite of different given input prices.

In short, Debreu and Farrell's papers, based on Koopman's (1951) and Pareto's (1897) foundations do a tremendous job in explaining the neoclassical production theory and the inherent inefficiency embedded in the system, by giving an intuitive yet sophisticated explanation for productive efficiency. Even though Farrell's technique is also far from perfect and limited in applicability, a brilliant set of ideas brought with it lead to a whole new world of nonparametric methods that not only compete with but also complete other statistical and econometric methods such as parametric and nonparametric regression analysis and linear modelling.

6. NON-PARAMETRIC FORM AND THE UNDESIRABLE OUTPUTS

The Modern DEA Models have been largely shaped by CCR (Charnes, Cooper, Rhodes, 1978) and BCC (Banker, Charnes, Cooper, 1984) papers based on the principles of Debreu (1951) and Farrell (1957), which today comprises over 70% of the studies using a non-parametric production function (Varabyova et. al., 2013). One of the major assumptions in the basic DEA Model is the "free disposability of outputs", which, although is a fair assumption, may sometimes cause problems. This becomes evident with "undesirable outputs" such as "environmental pollution" (Pittman, 1981), where free-disposability may not be feasible. This requires "*asymmetric treatment*" of desirable and undesirable outputs, which was not possible in Farrell's Technique (Pitmann, 1983).

The FGLP89 (Färe, Grosskopf, Lovell & Pasurka, 1989) and FGLY93 (Färe, Grosskopf, Lovell & Yaisawarng, 1993) papers distinguish between weak and strong disposability. Weak Disposability requires a reduction in "desirable output" for a proportional reduction in the "undesirable output", thus is not costless. In other words, producing more of desirable output requires a higher production of undesirable output;

therefore the efficiency is then tied to the minimum production of this "undesirable output" for the maximum production of desirable outputs, using the minimum level of inputs.

The FGLY93 Methodology, which aims to credit for the desirable output and penalize for the undesirable output, comes up with a Hyperbolic Efficiency Measure, which improves on the Enhanced Productivity Measure that prevailed before (FGLP89), by using the asymmetric efficiency measures, also capturing the shadow prices, rather than just relying on the market prices. Also, unlike the traditional method, the FLGY93 Technique enables us to have several efficiency measures depending on what to maximize/minimize, keeping what variables constant. In addition, there are 2 significant findings noted:

a) *Efficiency measures are very sensitive to the inclusion of undesired outputs*. The efficiency figures will be significantly different due to weak disposability and environmentally conscious firms will seem less efficient unless adjusted for the given regulations and/or conditions in place.

b) *Strong vs. Weak Disposability implies large costs to environmental precaution and regulations*, which is subject to a trade-off with respect to social marginal benefit. Along with the associated political concerns, it can be used as an argument for environmental policies.

7. COMPARING DEBREU-FARRELL WITH FGLP89 & FGLY93 PAPERS

Farrell's seminal methodology relies heavily on the free disposability of outputs, which, realistic as it may be, proves to be problematic with respect to undesirable outputs, which may not be freely disposable at all. This leads to a whole literature in DEA that differentiates between strong and weak disposability. In this context, weak disposability requires a reduction in good/desirable output for a proportional decrease in undesirable/bad output, thus it is costly.

Unlike Farrell's Technique, where all outputs are good outputs and tried to be maximized, the new Technique aims to credit for good output and penalize for bad output. However, this requires asymmetric treatment of good and bad output efficiency measures for proportional changes while this was not possible in the seminal Farrell's Technique, which employed a unique efficiency measure instead.

Conventional Efficiency Measure (Farrell), while ignoring undesirable outputs, aims

- Maximum proportionate increase in outputs
- Maximum proportionate decrease in inputs

Enhanced Efficiency Measure, (ignoring weak disposability), aims

- Maximum proportionate increase in desirable outputs
- Maximum proportionate decrease in undesirable outputs
- Maximum proportionate decrease in inputs

Hyperbolic Efficiency Measure (FGLY93) finally aims the same proportionate changes above with asymmetric efficiency changes to capture the effects of weak disposability. There are two major differences between the FGLP89 Study with Multilateral Productivity and the FGLY93 Study with Hyperbolic Efficiency:

First, FGLY93 uses shadow prices instead of market prices, which might be different for any firm, and do not constitute an inherently reliable source of efficiency evaluation. However, Shadow Prices, which are equal to their idiosyncratic Marginal benefits that vary by producers, are a much better fit and source of evaluation for efficiency. This is also a huge improvement on the original Efficiency Technique, first employed by Farrell.

Secondly FGLY93 uses multiple efficiency measures, rather than a single measure, depending on the what to maximize/minimize and keep constant. This does not only provide a huge flexibility and variety over the previous method, but also tends to produce more realistic efficiency figures.

8. SUMMARY AND CONCLUDING REMARKS

In the 1821 Ricardian framework of the classical era, labor is ever abundant, homogenous in production and constant in value. Rents only exist due to the scarcity of land, and capital accumulation is only possible only due to the left over profits after the distribution of (subsistence) wages and rents. Therefore production is a function of labor only.

The 1928 Cobb-Douglas paper of the neoclassical era shows, however, capital also plays a great role in the production, having a two way relationship with labor, which renders production a function of both labor and capital in addition to a third exogenous factor, called “Total Factor Productivity”, accounting for the level of technology among others while the economic thought shifts from averages to marginals, and from macro to micro.

Debreu (1951) and Farrell (1957) of the modern era point out that it is wrong to assume that all resources are used efficiently and the economy operates at the frontier, suggesting that the economy is not necessarily operating on the pareto-optimum frontier but usually inside at suboptimal levels due to underutilization of resources, inefficiency of economic organization and technical inefficiency. They suggest various measures for technical inefficiencyand ways to integrate this into the production function.

CCR (1978) and BCC (1984) papers formalize the methodology for non-parametric Data Envelopment Analysis based on Debreu and Farrell’s studies, which are later extended by FGLP89 and FGLY93 papers to allow for weak and strong disposability to include undesirable outcomes in the analysis as well as shadow prices instead of market prices.

There are certainly further developments in the literature which are beyond the scope of this paper such as the integration endogenous technology (Romer, 1986), non-parametric regression analysis (Meeusen & Broeck, 1977) and inclusion of the stochastic shocks in the production function (Kumbhakar & Lovell, 2000), and finally the admixture of stochastic and non-parametric methods (Ray & Kumbhakar, 2015), all of which we hope to address in future papers.

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